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Approximate analytical multiple solutions of the boundary layer flow over a shrinking sheet with power-law velocity

Baoheng Yao [a,b,*], Guoxiong Wu [c]

[a] State Key Laboratory of Ocean Engineering, Shanghai 200240, China
[b] Institute of Oceanology, Shanghai Jiaotong University, Shanghai 200240, China
[c] Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, UK

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ABSTRACT

The boundary layer flow over a shrinking sheet into a slot with power-law velocity is analytically studied by a newly developed technique namely homotopy analysis method (HAM). The present work provides analytically new solution branch in different solution areas with the aid of an introduced transformation. The analytical results show that quite complicated behaviors controlled by mass transfer parameters f_0 exist, including the known algebraically decaying solution, additional dual solutions and unique solution, which greatly differ from the continuously stretching surface problem. The new analytical solution branch enriches the solution family of the boundary layer flow over a shrinking sheet into a slot with power-law velocity, and helps to understand it deeply.

1. Introduction

Investigation of the boundary-layer flows of incompressible fluid over a stretched surface has important applications in electrochemistry and polymer processing. Since the pioneering work by Sakiadis [1], such researchers as Crane [2], Banks [3], Gupta and Gupta [4] and et al. have extensively analyzed the boundary layer flow on a continuously stretching surface. Liao [5,6] has found a new solution branch for both impermeable and permeable stretching sheets by means of the homotopy analysis method, a newly developed technique, showing the existences of multiple solutions for the stretching surfaces under certain conditions.

Recently, a quite interesting but different physical background from the case of stretching sheet, however, the flow over a shrinking sheet, is investigated by Miklavcic and Wang [7], indicating that mass suction is necessarily required to maintain the flow over the shrinking sheet. It is essentially a backward flow as discussed by Goldstein [8], which has exhibited quite distinct physical phenomena of interests from the forward stretching flow. Fang [9] has extended the shrinking sheet problem to a more general situation with a power-law velocity of the sheet shrinking into a slot, within which the effects of the wall drag and flow behaviors are also numerically studied. Greatly different from the continuously stretching sheet problem, some more complicated behaviors within the shrinking sheet are found.

Generally speaking, many nonlinear problems may admit multiple solutions, however, it is not easy to gain all the multiple solutions of a considered nonlinear problem by numerical techniques, and even harder for analytical techniques. Since there are quite few publications on boundary layer flow over a shrinking sheet, it is naturally an important topic whether it is possible to gain analytically both these multiple solutions and unique solution for the flow over the sheet shrinking into a slot with a power-law velocity. On one hand, the analytical results can enrich the solution family for the problem considered, and on the other hand, the analytical results can also be compared with the known numerical results for verification.

In this paper, therefore, inspired by the work mentioned above, the authors aim to investigate analytically the boundary layer flow over a shrinking sheet into a slot with power-law velocity. The present work provides analytically new solution branch of the problem considered in different solution areas, including both multiple solutions and unique solution by homotopy analysis method. The analytical results show that quite complicated behaviors controlled by mass transfer parameters f_0 exist, including the known algebraically decaying solution, and additional dual solutions and unique solution, which greatly differ from the continuously stretching surface problem. The new analytical solution branch enriches the solution family of the boundary layer flow over a shrinking sheet into a slot with power-law velocity, and helps to understand it deeply.

* Corresponding author at: Institute of Oceanology, Shanghai Jiaotong University, Shanghai 200240, China. Tel.: +86 21 34207293.
E-mail address: yaobaoheng@sjtu.edu.cn (B. Yao).

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2. Mathematical formulation

A steady, two-dimensional laminar flow over a continuously shrinking sheet in a quiescent fluid can be described by [1]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \quad (2)$$

subject to the boundary conditions

$$u(x,0)=-U_0x^m, \quad v(x,0)=v_w(x), \quad u(x,\infty)=0, \quad (3)$$

where u and v are the velocity components in the x and y directions respectively, and ν is the kinematic viscosity. Define $\psi(x,y)$ the stream function, and under the similarity transformation of

$$\psi(x,y)=\sqrt{\frac{2U_0\nu}{m+1}}x^{(m+1)/2}f(\eta), \quad \eta=\sqrt{\frac{(m+1)U_0}{2\nu}}x^{(m-1)/2}y, \quad (4)$$

the original equations become

$$f'''+ff''-\beta f'^2=0, \quad (5)$$

with the boundary conditions

$$f(0)=f_0, \quad f'(0)=-1, \quad f'(+\infty)=0, \quad (6)$$

where f_0 is the wall mass transfer parameter, and $\beta=\frac{2m}{m+1}$ when $-1<m\leqslant\infty$, $-\infty<\beta\leqslant 2$.

It is found by Fang [9] that the Eq. (5) has a special solution with algebraically decaying property, $f(\eta)=\frac{6/(2-\beta)}{\eta+\sqrt{6/(2-\beta)}}$ at $f_0=\sqrt{6/(2-\beta)}$. Moreover, the accurate solutions of Eq. (5) with some special parameters as $\beta=-1$ and $\beta=-2$, have been also given. As for other cases, numerical technique has to be used to solve the boundary layer problem considered. Therefore, it is naturally a challenging problem whether it is possible to gain analytically both these multiple solutions and unique solution for the flow over the sheet shrinking into a slot with a power-law velocity.

3. Series solution to the boundary layer flow over a shrinking sheet with power-law velocity by HAM

Under the transformation

$$f(\eta)=f_0+s(z)/\delta, \quad z=-\eta/\delta, \quad (7)$$

where $\delta<0$ is an introduced unknown constant to be determined later.

Thus, Eq. (5) becomes

$$s'''(z)-f_0\cdot\delta\cdot s''(z)-s(z)s''(z)+\beta s'^2(z)=0, \quad (8)$$

with the boundary conditions

$$s(0)=0, \quad s'(0)=\delta^2, \quad s'(+\infty)=0. \quad (9)$$

3.1. Zero-order deformation equation of HAM

Due to the boundary conditions in (9) by homotopy analysis method, a set of basis function

$$\{e^{-kz},\ k\geqslant 0\}, \quad (10)$$

is chosen to express the solutions of Eq. (8) as

$$s(z)=\sum_{k=0}^{+\infty}a_ke^{-kz}. \quad (11)$$

where a_k is a coefficient to be determined later. Thus, the solution expression of $s(z)$ which satisfies the boundary conditions (12) is provided.

According to the solution expression in (11) with the boundary conditions (9), it is directly led to choose the initial guess solution of Eq. (8) as

$$s_0(z)=\delta_0^2-\delta_0^2e^{-z}. \quad (12)$$

Moreover, an auxiliary linear operator L is chosen to be

$$\mathrm{L}[\phi(z;q)]=\frac{\partial^3\phi}{\partial z^3}-\frac{\partial\phi}{\partial z}, \quad (13)$$

which has the property of

$$\mathrm{L}[C_1e^{-z}+C_2+C_3e^z]=0, \quad (14)$$

where C_1, C_2 and C_3 are integral constants to be determined by the boundary conditions. Eq. (8) also suggests that a nonlinear operator N should be defined as

$$\mathrm{N}[\phi(z,q),\Lambda(q)]=\frac{\partial^3\phi(z,q)}{\partial z^3}-f_0\Lambda(q)\frac{\partial^2\phi(z,q)}{\partial z^2}-\phi(z,q)\frac{\partial^2\phi(z,q)}{\partial z^2}+\beta\left[\frac{\partial\phi(z,q)}{\partial z}\right]^2, \quad (15)$$

where $q\in[0,1]$ is an embedding parameter.

By introducing a non-zero auxiliary parameter $\hbar$, zero-order deformation equation of **HAM** is constructed as

$$(1-q)\mathrm{L}[\phi(z,q)-s_0(z)]=q\hbar\mathrm{N}[\phi(z,q),\Lambda(q)], \quad (16)$$

with the boundary conditions

$$\phi(0,q)=0, \quad q\Lambda^2(q)+(1-q)\delta_0^2-\left.\frac{\partial\phi(z,q)}{\partial z}\right|_{z=0}=0, \quad \left.\frac{\partial\phi(z,q)}{\partial z}\right|_{z=+\infty}=0. \quad (17)$$

Clearly, when $q=0$ and $q=1$, the solution of Eq. (8) is given respectively by

$$\phi(z,0)=s_0(z), \quad \Lambda(0)=\delta_0, \quad (18)$$

and

$$\phi(z,1)=s(z), \quad \Lambda(1)=\delta, \quad (19)$$

where δ_0 is the initial approximation of δ. Thus, $\phi(z,q)$ varies continuously from the initial guess solution of Eq. (8), $s_0(z)$, to the exact solution $s(z)$, as q increases from 0 to 1, and so does $\Lambda(q)$. Therefore, $\phi(z,q)$ and $\Lambda(q)$ can be expanded regarding to q in Taylor series to produce

$$\phi(z,q)=s_0(z)+\sum_{k=1}^{+\infty}s_k(z)q^k, \quad (20)$$

and

$$\Lambda(q)=\delta_0+\sum_{k=1}^{+\infty}\delta_kq^k, \quad (21)$$

where

$$s_k(z)=\frac{1}{k!}\left.\frac{\partial^k\phi(z,q)}{\partial q^k}\right|_{q=0}, \quad \delta_k=\frac{1}{k!}\left.\frac{\partial^k\Lambda(q)}{\partial q^k}\right|_{q=0}. \quad (22)$$

Provided that the auxiliary linear operator L, the initial guess solution $s_0(z)$, auxiliary parameters $\hbar$ are properly chosen so that the convergence of the above series is guaranteed at $q=1$, the series solution of Eq. (8) can be expressed as