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Analytical solution to non-Fourier heat conduction as a laser beam irradiating on local surface of a semi-infinite medium

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ABSTRACT

The two dimensional non-Fourier heat conduction in a semi-infinite medium on which local surface is irradiated by a laser beam is analyzed. The mathematical model is based on hyperbolic heat conduction equation with thermal relaxation time and is deals with boundary condition with time step heating and uniform temperature distribution in local area of the surface. A new analytic solution to the problem is derived by using Laplace and Hankel transforms and is owing to finding analytic result of inverse Laplace transform to time domain exactly. The solution of the temperature field in semi-infinite medium is expressed as an infinite integral of known function. The proposed numerical technique to the integral is implemented. Non-Fourier effect of temperature field is shown in numerical examples. The evolution of contour plots of temperature field over time is presented by evaluating the analytic solution demonstrated significant difference between hyperbolic and classical heat conduction.

1. Introduction

Laser irradiation technology to material surfaces has been widely used in engineering such as hardening and cladding of metal surface [1]. Also, it can be used in modern medicine such as laser surgery operation [2]. In these cases, the local surface of metallic materials and biological tissues are suddenly heated by laser beam, and the temperature in the inner region near the surface irritated could rise in a very short time. To be able to understand features of heating conduction arising as laser irradiation, we must make certain of temperature field. So, modeling the heating process is of great importance. Since laser irradiation will lead to very large heat fluxes applied in an extremely short time window and temperature gradient is extremely large, then the wave form propagation of heat in media becomes dominant [3], which is far different from the diffusion nature in the classical heat conduction problem. In such situations, the classical parabolic heat conduction equation based on Fourier law could not fully model the thermal transient behavior of media. A modified theory called hyperbolic heat conduction has been used to investigate heat propagation problems with non-Fourier effect, like those occurring in laser irradiation [4–6]. During the last four decades, considerable research has been carried out to solve hyperbolic heat conduction problems for different geometry and boundary conditions. So far a few analytical solutions to the problems are available in the literature, however, most of them are restricted to solve one-dimensional hyperbolic heat conduction problems. For example, Ready [7] introduced the analytic solution for the temperature rise inside a substrate due to a step input pulse. Baumeister and Hamill [8] presented a solution of temperature field in case that the temperature on entire surface of a semi-infinite body is stationary. Yilbas [9] studied the temperature response in a semi-infinite solid due to a laser pulse varying exponentially in time, and the convective boundary condition case was further considered by Yilbas and Kalyon [1]. For spherical symmetry cases, Shirmohammadi and Moosaie [10] derived an analytical solution for a hollow sphere exposed to a periodic boundary heat flux. Recently, Ahmadikia et al. [11] solved analytically the thermal wave problem during laser irradiation of skin tissue. Since the actual laser beam irradiation concentrates in *local* area of surface of material, spatial distribution of temperature inside medium should be multidimensional. In multidimension cases, to find analytical solution to heat conduction problem is still a challenge work. On the other hand, since analytical solutions can provide useful information to reduce the experimental time and cost, analytical investigations into the heat conduction problem are valuable. For finite medium like wall, plate and cubic, Chen [12] provided a number of solutions to multidimensional hyperbolic heat conduction problem.

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Nomenclature

a	radius of the laser beam
c_0	specific heat capacity of the material (J/(kg K))
k_0	coefficient of heat conduction (W/(m K)
r	space variable (m)
T_c	temperature of the surface caused by laser beam (K)
t	time variable (s)
e	natural logarithm base
z	space variable (m)
x, y, z	coordinate (m)
s	Laplace transform parameter
$\mathcal{L}$	Laplace transform
$\mathcal{L}^{-1}$	inverse Laplace transform
$\mathcal{H}_0$	Hankel transform of the zero order
$\mathcal{H}_0^{-1}$	inverse Hankel transform of the zero order
H	Heaviside function
J_0	Bessel function of the zero order
g	Function names used to simplify the deduction of formulas
J_1	Bessel function of the 1st order
T	temperature

Greek letters

ρ_0	the density of the material (kg/m^3)
ξ	dimensionless variable
γ	a dimensionaless number, $\gamma = \rho_0 c_0 / k_0$
φ	angle
τ_0	relaxation time of a material in the conducting progress

Superscript

$\tilde{T}$	Laplace transform of the temperature function T with respect to time variable t
$\hat{T}$	Hankel transform of the function $\tilde{T}$ with respect to radius variable r

The purpose of this work is to provide an analytic solution to temperature response in a semi-infinite medium when finite circle area of the medium surface is uniformly and suddenly heated by a laser beam irradiation. On the basis of classical heat conduction equation, the exact solution of the problem was given in [13]. In this article, the non-Fourier effect is considered and a new analytic solution for the hyperbolic heat conduction equation of the problem is constructed by using Laplace and Hankel transform method.

2. Mathematical modeling and analytic solution

2.1. Mathematic description of the problem

Consider the hyperbolic heat conduction problem arising as a laser beam irradiating on the local surface of a semi-infinite heat transfer medium. The laser beam irradiates vertically and with homogeneous distribution into a circle area of radius a on the medium surface. By using cylinder coordinate system and in view of the axisymmetric property of the problem, we suppose the temperature field T does not depend on the angle φ (see Fig.1), that is, $T = T(r, z, t)$ $(r \geq 0,\ z \geq 0,\ t \geq 0)$. To describe the instantaneous response of laser irradiation, we employ the hyperbolic heat conduction equation as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \gamma\left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}\right) \tag{1}$$

where the parameter $\gamma = \rho_0 c_0 / k_0$

As the irradiated depth of the medium is considerable small [1], we suppose the laser beam temperature T_c in the circle area of the

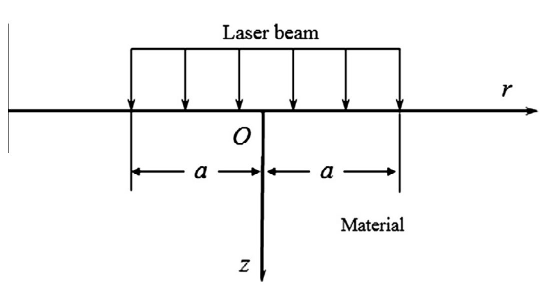

Fig. 1. A laser beam irradiates on the local circle area of semi-infinite medium surface.

surface is known, and the temperature is step jumping in initial time and uniform destribution in the the circle area, then the boundary condition can be written as:

$$T(r, 0, t) = T_c H(t) H(a - r) \quad (t \geq 0,\ r \geq 0) \tag{2}$$

In the z-coordinate axis, the axisymmetric condition gives

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \tag{3}$$

The temperature at infinity satisfied the following conditions

$$\lim_{r\to+\infty} T = \lim_{r\to+\infty} rT = \lim_{r\to+\infty} r\frac{\partial T}{\partial r} = 0 \tag{4}$$

$$\lim_{z\to+\infty} T = 0 \tag{5}$$

The initial condition is

$$T(r, z, 0) = \frac{\partial T}{\partial t}(r, z, 0) = 0 \tag{6}$$

In mathematical, the problem is reduced to solving Eq. (1) with the conditions (2)–(6).

2.2. Analytic solution in the Laplace and Hankel-transformed domain

Performing the Laplace transformation on Eq. (1) with respect to t and using the initial condition (6), we have

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\tilde{T} + \frac{\partial^2 \tilde{T}}{\partial z^2} = \gamma s(1 + \tau_0 s)\tilde{T} \tag{7}$$

The Laplace transformation to the boundary condition (2) is

$$\tilde{T}(r, 0, s) = \frac{T_c}{s} H(a - r) \tag{8}$$

The conditions (3), (4) become

$$\left.\frac{\partial \tilde{T}}{\partial r}\right|_{r=0} = 0, \quad \lim_{r\to+\infty} \tilde{T} = \lim_{r\to+\infty} r\tilde{T} = \lim_{r\to+\infty} r\frac{\partial \tilde{T}}{\partial r} = 0 \tag{9}$$

$$\lim_{z\to+\infty} \tilde{T} = 0 \tag{10}$$

The Hankel transformation on the first term in the left side of Eq. (7) is as follows: