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Effect of “the thermal piston” in a dynamic thermoelastic problem

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ABSTRACT

There are experimental works demonstrating distinction of initiated thermal stresses in dielectrics and metals at short-term laser heating. However, such thermal stresses in metals are not described by analytical solutions obtained within existing models of dynamic thermoelasticity.

In this paper a dynamic thermoelasticity problem with short pulse laser heating is analyzed. The difference between mechanisms of heat conduction in dielectrics and metals is taken into account. The analysis is based on the model of “the thermal piston” moving due to the flow of free electrons transferring heat in metals. This model may be associated with hydrodynamic problem of piston motion.

The additional contribution to the stress pulse in heat-conductive media (metals) is taken into account by solving the wave equation with a moving boundary. It is shown that such approach describes well the difference in the parameters of the stress pulses in dielectrics and metals observed in the experiments. As a result, it is found that the average mechanical stress pulse is not zero, and fast non uniform heating can lead to moving of metal objects.

The presented results of experiments and the analysis of dynamic thermoelasticity problem confirm the possibility of movement of heat-conducting objects (metals) under pulsed non uniform heating. The object movement is determined by the stretching phase of thermoelastic stresses, the formation of which is caused by the movement of “the thermal piston”.

1. Introduction

The dynamic problem of thermoelasticity was first considered by Danilovskaya [1] who found a solution for stress during thermal blow. Later, a more exact solution for the problem of thermal stress, due to a pulsed thermal flow directed to a semi space border, was given in [2].

Subsequent studies of the dynamic problem of thermoelasticity, including an analysis of the effect of connectivity, boundary and initial conditions [3–6], as well as the use of the Cattaneo, CTE, L–S and G–L models [7–11], have not made any fundamental changes in the solution for thermoelastic stresses. Temporal profiles of stress pulses (or a mass velocity of particles) described by these solutions, represent a bipolar compression–tension pulse with commensurate values of the amplitudes and the duration of the phases comparable with the duration of a radiation pulse. In this case, the thermal conductivity of the medium has a rather weak effect on the change of the amplitude and the duration of the phases.

However, it was noted in a number of experimental works [12–14] that the waveform of stress pulses in metals deviates considerably from that predicted by the above-mentioned solutions. Fig. 1 shows the normalized dependences of the pulses of thermal stresses in dielectrics and metals [14]. A fundamental difference in the ratio of the durations and the magnitudes of phases of compression and extension of the thermal stresses in dielectrics and metals is obvious. That is, in contrast to the classical solutions [1,2] and the solutions of the Cattaneo, CTE, L–S and G–L theories [7–11], an average mechanical impulse in metals is not equal to zero. These results suggest that heat and mass transfer in metals are common consequence of the movement of nearly free electrons.

The above necessitates gaining a deeper insight into the formation of the dynamic thermoelasticity problem for thermo- and electrical conductivity mediums, considering the mathematical and physical aspects of the problem, as well as experimental results [12–14], which indicate an essential distinction in the formation of thermoelastic tension stresses for heat-conducting and not heat-conducting mediums.

It is known that electrons carry a large part of heat in metals and the flow of these electrons can be described by the hydrodynamic model [15,16]. In the heat transfer process by means of electrons, each electron under the influence of “thermal force” $E \sim k\Delta T$

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carries heat energy kT (k is Boltzmann's constant). On the other hand, an-harmonic expansion terms of crystal potential energy expanded into the Taylor series according to the lattice shift are connected with thermal expansion. According to the Mie–Gruneisen equation of state an increase in internal energy of the micro volume leads to the generation of elastic waves. Thus, the heat transfer process by means of electrons looks like a "heat piston".

2. Problem statement

Let us consider the problem of one-dimensional dynamic thermoelasticity using the "thermal piston" model. The standard equations of the disconnected dynamic thermoelasticity problem for a one-dimensional case and Duhamel's ratio can be written in the form [1]:

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \beta\frac{\partial T}{\partial x} = 0$$

$$\frac{\partial T}{\partial t} - \chi\frac{\partial^2 T}{\partial x^2} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial x} = \frac{\sigma}{\lambda + 2\mu} + \beta T$$

where $\beta = (3\lambda + 2\mu)\alpha_T/(\lambda + 2\mu)$; $c^2 = (\lambda + 2\mu)/\rho$; λ, and μ are Lame's constants; α_T is the thermal expansion coefficient; ρ is the density; χ is the thermal diffusivity coefficient; u is the displacement; and T is the temperature of the body.

The initial conditions are

$$T(x,0) = 0; \quad \frac{\partial u(x,0)}{\partial x} = \frac{\partial^2 u(x,0)}{\partial x \partial t} = 0 \qquad (1a)$$

The boundary conditions are

$$T(0,t) = T_0; \quad \frac{\partial u(0,t)}{\partial x} = 0 \qquad (1b)$$

Fig. 2(a) shows the characteristics $X = f(t)$ of the equation system (1). The intersection point of the characteristics $(x_* = \chi/c, t_* = \chi/c^2)$ is determined by the thermal and elastic properties of the materials. In addition, this point defines the boundary of possibility of application of the continuum mechanics equations to the analysis of the thermoelastic effects.

For example, the intersection point of the characteristics for metals corresponds to the values of $t_* \approx 10^{-12}$–10^{-11} s and $x_* \approx 10$–100 nm. However, the evaluation time of the electron–ion interaction gives values of $\tau_{e-ph} \approx 10^{-11}$–$10^{-10}$ s, and the evaluation time of the ion–ion interaction (central collisions) gives values of $\tau_{ph-ph} \approx 10^{-10}$–$10^{-9}$ s. That is, until the characteristic point (x_*, t_*), it is incorrect to use the concept of the uniform thermodynamic temperature of the medium, as well as the approach of the theory of elasticity.

Note that, the hyperbolic Catteneo model of heat conductivity is widely applied to describe phenomena at large temperature gradients, when the classical correspondence between heat flux and the gradient is not performed. For example, the Catteneo model is usually used to describe the processes of heat shock, when the duration of the thermal loading is less than the relaxation time of the heat flux τ_q [9]. For metals, an estimation of this time gives the value $\tau_q \approx 10^{-11}$ s [4,10]. The duration of the laser pulse in our experiments is $t_p \approx 3 \times 10^{-8}$ s.

Thus, a correct analysis of a thermoelastic response of solids, based on (1), is possible only for the time $t \geqslant t_*$ and with the exclusion of the formalism of the heat equation (which leads to an infinite speed of heat distribution).

The dependences of temperature calculated according to the solution of the thermal equation [14] for radiation on the metal with the duration of a laser pulse of $t_p \cong 3 \cdot 10^{-8}$ s are presented in Fig. 2(b) and (c). The change in temperature with depth X for different values of the dimensionless time t/t_p is shown in Fig. 2(b). The temperature dependences on the dimensionless time t/t_p for the different distances from the irradiated surface are shown in Fig. 2(c).

The analysis of the temperature change of the medium in space and time (Fig. 2(b) and (c)) indicates the need to consider the contribution of thermal stresses of micro volumes of the medium heated by the spread of the heat flux (Fig. 2(d)).

Thereby, such generation of elasticity waves in the thermal flow movement due to the electron transport process can be associated with the hydrodynamic problem of piston moving (in our case the "thermal piston").

To account for the thermal piston influence in the case of $t > t^*$ we will present (1) as follows:

$$u = u_\alpha + u_T \qquad (2)$$

where u_α is displacement in accordance with (1) with the corresponding usual initial and boundary conditions [2] and u_T is the displacement due to the "thermal piston" activity.

In the case of $u_T \to 0$, we have the ordinary equation system for non-thermal conductivity materials. For heat-conducting media (metals), we obtain a wave equation for U in addition to (1):

$$\frac{\partial^2 u_T}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u_T}{\partial t^2} = 0 \qquad (3)$$

Wave equation (3) for u_T describes a process due to thermal transformation in the thermal-transfer medium and it requires the setting of boundary conditions for the characteristics of the thermal equation $x(t) = 2\sqrt{\chi t}$ [17], and for time $t \geqslant 2\sqrt{\chi t}/c$ the heated medium range is unloaded; consequently, there is no stress on the boundary $\sigma_x = 0|_{x=2\sqrt{\chi t}}$.

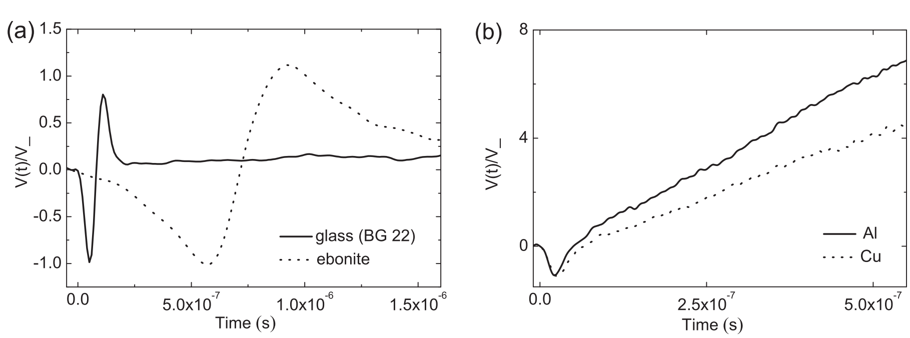

Fig. 1. Time dependences of pulses of thermal stresses initiated by a laser pulse of $t_p \cong 3 \cdot 10^{-8}$ s: (a) in dielectrics, and (b) metallic samples [14].