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A three-dimensional simulation of transient natural convection in a triangular cavity

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ABSTRACT

The three-dimensional (3D) numerical simulation of natural convection in a triangular cavity with the top cooling and the bottom heating is performed and compared with the previous experiment. The numerical results show that the development of natural convection in the cavity following an initially isothermal and stationary state may be classified into three stages: an initial stage, a transitional stage and a quasi-steady stage, which is consistent with the experimental result. Natural convection flows in each of the stages, particularly the 3D flow structure such as longitudinal rolls, are described. Further, the regimes of transient natural convection are discussed using a simple scaling analysis and the scaling relations obtained are validated by the present numerical results.

1. Introduction

Natural convection in a triangular cavity is present in many domestic and industrial systems. One application attracting a great deal of attention is heat transfer through natural convection in attics of buildings because reasonable design and construction of houses can provide thermal comfort for occupants. In particular for an energy-conscious society, energy consumption through heating and air-conditioning should be minimized. Accordingly, natural convection flows and heat transfer in the triangular cavity have been paid increasing attention over the last three decades [1].

One of the earliest studies of natural convection in an attic space was performed by Flack [2]. The previous studies focus on two major sets of temperature boundary conditions: night time cooling (cooling the top and heating the bottom) and day time heating (heating the top and cooling the bottom). It has been demonstrated that natural convection flows in the triangular cavity usually remain stable and laminar in the daytime condition but the transition from laminar to turbulent flows may occur for a large Rayleigh number in the night-time condition [1–4].

Flack [2] first investigated natural convection and heat transfer in the triangular cavity for the attic problem in the day-time boundary condition. It has been found that natural convection flows in the cavity are laminar for the Rayleigh number of up to 4.9×10^7 for which heat transfer through the cavity is dominated by conduction. Further studies (see e.g. [5]) have also shown that natural convection flows, although considerably weaker, evolve from a two-cell to a multiple-cell flow pattern dependent on the Rayleigh number and the aspect ratio. Apart from the flow pattern, the flow development with time under different dynamical regimes is also interesting and has been recently investigated using a scaling analysis. The scales in different conditions for the day-time attic problem have been obtained [6–9]. Additionally, the dependence of the Nusselt number on the Rayleigh number (or the Grashof number) and the aspect ratio has been quantified [10–12].

There are a greater number of studies which focus on the night time attic problem Poulikakos and Bejan [13] experimentally investigated the night-time attic problem for high Rayleigh numbers and their results are consistent with the measurements of Flack [2–4]. Earlier studies of the attic problem usually assumed that natural convection flows in an attic space were symmetrical about the vertical mid-plane. Therefore, natural convection flows in a half section of an isosceles triangular cavity were considered in which an adiabatic condition is applied for the vertical mid-plane in the numerical and analytical study (see [14]). However, it has been demonstrated that as the Rayleigh number increases, the symmetrical flow in the isosceles triangular cavity is destroyed; that is, the transition from symmetrical to asymmetrical flow structures occurs in the night time condition if the Rayleigh number is larger than a critical value [15,16]. In fact, this

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Nomenclature

A	aspect ratio
g	acceleration due to gravity
H, L, W	height, half length and width of the cavity
k	thermal conductivity
l	length of the inclined wall
n	coordinate normal to the wall
Nu	Nusselt number
p	dimensionless pressure
Pr	Prandtl number
Q^*	flow rate
Ra	Rayleigh number
s	coordinate parallel to the wall
S	area of the wall
t	dimensionless time
T	dimensionless temperature
t^*	time
T^*	temperature
T_0	initial temperature
T_c, T_h	temperatures of the cold inclined wall and the hot bottom
u, v, w	dimensionless x-, y- and z-velocity
u_w	dimensionless velocity of the intrusion flow
u_w^*	velocity of the intrusion flow
U_x, U_y, U_z	averaged x-, y- and z-velocity
V	volume of the cavity
v_w	dimensionless velocity of the thermal boundary layer adjacent to the inclined wall
v_w^*	velocity of the thermal boundary layer adjacent to the inclined wall
x, y, z	dimensionless coordinate

Greek symbols

β	coefficient of thermal expansion
δ	dimensionless thickness of the thermal boundary layer adjacent to the inclined wall
δ^*	thickness of the thermal boundary layer adjacent to the inclined wall
Δ^*	thickness of the intrusion flow
Δt	dimensionless time step
ΔT	temperature difference between the bottom and the inclined wall
κ	thermal diffusivity
ν	kinematic viscosity
ρ	density
τ_i	dimensionless time scale at which the viscous term balances the advection term in the intrusion flow
τ_i^*	time scale at which the viscous term balances the advection term in the intrusion flow
τ_s	dimensionless time scale at which convection balances conduction in the thermal boundary layer adjacent to the inclined wall
τ_s^*	time scale at which convection balances conduction in the thermal boundary layer adjacent to the inclined wall

asymmetrical flow is one of two possible mirror image asymmetric solutions due to the occurrence of a supercritical pitchfork bifurcation for the Rayleigh number above a critical value (see [15] for details). Furthermore, some fundamental scales of transient natural convection flows for the night-time attic problem have been obtained based on a scaling analysis [8,14].

Although natural convection flows for the night-time attic problem were experimentally and numerically investigated, the development of natural convection flows in the triangular cavity following sudden cooling of the top and heating of the bottom was paid less attention until Lei et al. [17]. The shadowgraph experiment with water as the working fluid was conducted for this study. Later on, the corresponding 2D numerical simulation was also performed by Lei et al. [18] and the development of natural convection flows was described based on the 2D numerical results. The 2D numerical simulation is unable to accurately describe an essentially three-dimensional flow mainly due to the presence of longitudinal rolls in the triangular cavity. However, the literature review shows that few investigators have paid attention to 3D numerical simulation, with the exception [19]. This study showed the transition from symmetric to asymmetric flow in a pentahedral space of isosceles triangle cross-section (as previously described, also see [15]) and demonstrated the presence of the 3D flow behavior. Accordingly, a 3D numerical simulation corresponding to the experiment in [17] has been motivated in this study. In order to observe the 3D flow structure of transient natural convection in the cavity (particularly longitudinal rolls), the full 3D governing equations have been directly solved. The 3D flow structure of natural convection in the triangular cavity is described in comparison with the previous experiment in [17].

In the rest of this paper, the governing equations and numerical procedures are described in Section 2; the development of natural convection in the triangular cavity at different stages is characterized in Section 3; natural convection and heat transfer are quantified and discussed using a simple scaling analysis in Section 4; and finally, the conclusions are presented in Section 5.

2. Numerical procedures

An isosceles triangular cavity was under consideration in this study. Fig. 1 shows a schematic of the computational domain of height (H), which is half of length ($L = 2H$) and half of width ($W = 2H$). Here, the dimensions of the cavity are the same as those in the experiment by Lei et al. [17]. In order to avoid the singularities at the intersection regions between the roof and the bottom, the tiny tips of approximately 4%L were cut at both sides of the geometric plane at $x = 0$. It is expected that this slight modification of the flow domain has an insignificant impact on the calculated flow and heat transfer (see [20–22]).

The 3D governing equations were normalized using the following scales: x, y, z $\sim H$; $t \sim H^2/(\kappa \mathrm{Ra}^{1/2})$; $(T - T_0) \sim (T_h - T_c)$; u, v, $w \sim \kappa \mathrm{Ra}^{1/2}/H$; and $\rho^{-1}\partial p/\partial x$, $\rho^{-1}\partial p/\partial y$, $\rho^{-1}\partial p/\partial z \sim \kappa^2 \mathrm{Ra}/H^3$. The development of natural convection in the cavity may be described by the 3D non-dimensional governing equations with the Boussinesq approximation [20]:

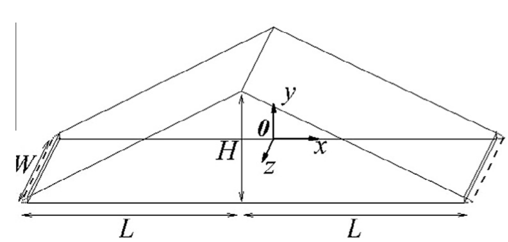

Fig. 1. Schematic of the triangular cavity.