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Mixed convection heat transfer in power law fluids over a moving conveyor along an inclined plate

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ABSTRACT

We study the mixed convection boundary layer heat transfer of power law fluid over a moving conveyor along an inclined plate. The effects of shear flow and power law viscosity on the temperature field are taken into account according to a modified Fourier law. Approximate analytical solutions are obtained by the homotopy analysis method (HAM). Results indicate that heat transfer is strongly dependent on the values of power law exponent, inclination angle, boundary velocity ratio and Prandtl number. Three distinct characteristics are found for power law exponents $0 < n < 1$, $n = 1$ and $n > 1$, especially the nonlinear behavior due to skin friction and local Nusselt number shown in Figs. 4 and 17, which has never been reported before. The decrease of inclination angle causes the loss of velocity boundary layer but the gain of temperature boundary layer. Heat transfer efficiency is enhanced but skin friction is diminished with the increase in velocity ratio (the ratio of conveyor velocity/mean velocity of flow field). Critical ratio (with skin friction zero) is obtained which strongly depends on the power law exponent. The effects of involved parameters on the velocity and temperature fields are analyzed.

1. Introduction

In recent years, the investigation of mixed convection heat transfer has attracted considerable attention in many fields of science and technology because of its wide applications, for example, the circulation due to different density along the vertical direction in a lake due to seasonal variation, different temperature atmospheric flow, and heat exchanger in fuel in nuclear reactors. The most prominent feature of mixed convection is buoyancy force caused by varying density and temperature. Mathematically speaking, under Boussinesq's hypothesis, the momentum and energy equations describing mixed convection are highly coupled. According to Prandtl's boundary layer theory, a fluid with mixed convection will induce a boundary layer close to the vertical plate due to the viscosity of fluid. Through dimensional analysis, boundary layer governing equations can be simplified by the Navier–Stokes equations. The air-heat convection around the vertical plate has been measured and the results of existence of momentum and thermal boundary layers have been proved [1].

It is well recognized that non-Newtonian fluids are important in science research and engineering. A main reason may be attributed to the fact that the fluids (such as molten plastics, pulps, slurries, emulsions), which do not obey Newtonian postulate that the stress tensor is directly proportional to the deformation tensor, are produced industrially in increasing quantities. Many models have been proposed to describe behaviors of such fluids. Among those models, the power-law model [2], in which shear stress varies according to a power-law function of strain rate, has gained considerable acceptance. The boundary layer equations for a power law fluid were studied in [3–7]. The flow and heat transfer in power law fluid over a stretching sheet were investigated in [8,9]. Zhang et al. [10] and Bharti et al. [11] solved numerically the thermal boundary layer on a continuous moving surface in a power law fluid. They asserted that boundary layer temperature distribution depends on not only plane velocity, but also on the power law exponent n and generalized Prandtl number N_{pr}.

In classical works for a power law fluid, power-law kinematic viscosity was introduced in the momentum equation but the energy equation was treated the same as in Newtonian fluids. Obviously, this is inconsistent with the fact that changing viscosity should affect both momentum and heat transfer. Some researchers have paid special attention to this inconsistency as the rough assumption of thermal conductivity for non-Newtonian fluid does not meet the sophisticated industrial requirements. Pop et al. [12–14] proposed a model for heat transfer that the thermal

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Nomenclatures

ρ	density of fluid	U	velocity component along x
c_p	specific heat capacity of fluid	V	velocity component along y
n	power law exponent	u, v	dimensionless velocity components
γ_u	velocity ratio coefficient	θ, w	dimensionless temperature
γ_c	critical velocity ratio coefficient	X	distance along the surface from the leading edge, x dimensionless distance
φ	inclined angle	Y	distance normal to the surface, y dimensionless distance
β	expansion coefficient	ψ	stream function,
L	characteristic length	f	dimensionless stream function
T	temperature	τ_{XY}	shear stress
T_w	surface temperature	Nu	local Nusselt number in power law fluid
T_∞	free stream temperature	Gr_{nx}	local Grashof number in power law fluid
U_∞	characteristic velocity		

conductivity of non-Newtonian fluids has power-law dependence on the velocity gradient. Zheng [15,16] took the effects of power-law viscosity on temperature field into account by assuming that the temperature field is similar to the velocity field. The Navier–Stokes equation and energy equation are modified with Fourier's-law heat conduction. They assumed that the thermal diffusivity varies as a function of velocity gradient or temperature gradient in energy equation of a power law fluid. A comparison for both models was presented. Natalia and Pop [17] investigated the steady mixed convection stagnation point flow over a vertical flat plate with a second order slip and the maintained heat flux. Two branches of solutions, upper and lower branch, were found in a certain range of mixed convection and velocity slip parameters. In addition, many other mixed convection heat transfer problems with different boundary conditions were investigated such as convection in inclined rectangular [18], mixed convection heat transfer with the effects of magnetic field [19,20] were also considered in porous medium and nanofluid [21–23].

Homotopy analysis method (HAM) was introduced by Liao in 1992, which has been successfully applied to many nonlinear problems, especially the calculation of boundary layer problems. Liao [24–29] proved that the series solution obtained by HAM convergence strongly depends on an auxiliary parameter h. In addition, some associated optimization and optimal homotopy-analysis methods were added recently by Liao [30]. The way as how to construct the initial guess solutions for natural convection with natural boundary conditions can be found in ref. [31].

The study for mixed convection heat transfer of non-Newtonian fluid, so far in our opinion, is inadequate. In this paper, we study mixed convection heat transfer of power law fluid over a moving conveyor along an inclined plate. Unlike most classical studies for Newtonian fluids, the effects of power-law fluid viscosity on temperature field are taken into account here by us by assuming that the temperature field is similar to velocity field with a modified Fourier's law. The effects of moving conveyor and inclined plate on the overall convection system, i.e., the velocity ratio coefficient $\gamma_u = \frac{U_w}{U_m}$ and incline angle φ, the generalized Prandtl number N_{pr} and power law exponent n on velocity and temperature field are graphically assessed, respectively. The generalized local Grashof number Gr_n and local Nusselt number Nu for power law fluids are also derived and discussed.

2. Governing equations

Consider a steady laminar mixed convection boundary layer flow and heat transfer in power-law fluid over a moving conveyor along an inclined plate, where the essential features of such a flow are illustrated in Fig. 1. It is assumed that the body force is $X = -\rho g$, and the boundary layer governing equations describing conservation of mass continuity, momentum and energy can be written as:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\bar{\mu}}{\rho}\frac{\partial}{\partial Y}\left(\left|\frac{\partial U}{\partial Y}\right|^{n-1}\frac{\partial U}{\partial Y}\right) + g\beta(T - T_\infty)\sin\varphi \tag{2}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\bar{\lambda}}{\rho c_p}\frac{\partial}{\partial Y}\left(\left|\frac{\partial U}{\partial Y}\right|^{n-1}\frac{\partial T}{\partial Y}\right) \tag{3}$$

$$Y = 0 : U = U_w,\ V = 0,\ T = T_w \tag{4}$$

$$Y \to \infty : U = 0,\ T = T_\infty \tag{5}$$

where the shear stress is characterized as $\tau = \bar{\mu}(|\frac{\partial U}{\partial Y}|^{n-1}\frac{\partial U}{\partial Y})$, the kinematic viscosity is $\bar{v} = \omega|\frac{\partial U}{\partial Y}|^{n-1}$($\bar{\mu}$ and $\omega = \bar{\mu}/\rho$ are positive constant), $\bar{\lambda}|\frac{\partial U}{\partial Y}|^{n-1}$ is the generalized thermal conductivity in terms of a modified Fourier's law, β is the expansion coefficient of power law fluid, U_w is the velocity of moving conveyor, T_w is the surface temperature of the inclined plate, $0 < \varphi \leqslant \frac{\pi}{2}$ is the angle between inclined plate with respect to the horizontal direction. The case $n = 1$ corresponds to a Newtonian fluid, $0 < n < 1$ is pseudo-plastic non-Newtonian fluids while $n > 1$ describes dilatant fluids, respectively.

The following dimensionless variables are introduced:

$$u = \frac{U}{U_m},\quad v = \frac{V}{U_m},\quad x = \frac{X}{L\sin\varphi},\quad y = \frac{Y}{L\sin\varphi},\quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

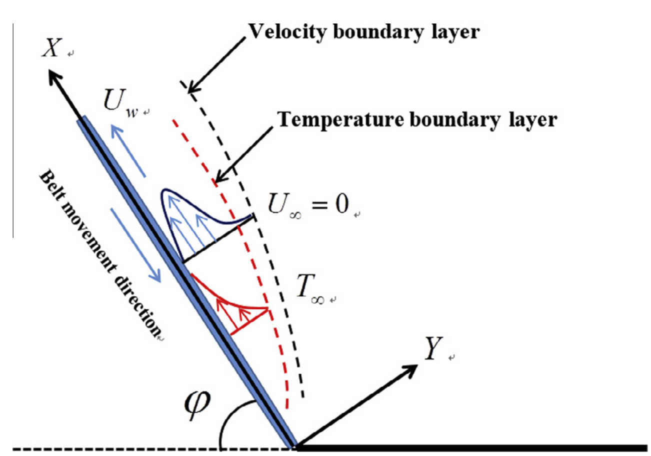

Fig. 1. Schematic showing physical model and coordinate system of inclined moving conveyor.