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Analytical study of third grade fluid over a rotating vertical cone in the presence of nanoparticles

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ABSTRACT

Present article deals with the study of third grade fluid flow over a rotating vertical cone in the presence of nanoparticles i.e. thermophoresis and Brownian motion. Solutions for the boundary layer momentum, energy and diffusion equations are carried out by a well-known analytical technique namely Homotopy analysis method. The interesting findings for essential physical parameters are demonstrated in the form of graphs and numerical tables. Also, a suitable comparison has been made with the prior results in the literature as a limiting case of the considered problem.

1. Introduction

In recent years the study of non-Newtonian fluid flows have attained noticeable importance due to its massive applications in many engineering and industrial processes. Non-Newtonian fluids with heat and mass transfer are important in processing of food, making of paper, lubrications with heavy oils and greases. Due to the practical significance of non-Newtonian fluids, several researchers have presented various non-Newtonian fluid models [1–3]. The third grade fluid model is one of the most substantial fluid models that display all the properties of shear thinning and shear thickening fluids. Ellahi et al. [4] studied generalized couette flow of a third-grade fluid with slip: the exact solutions. The influence of variable viscosity and viscous dissipation on the non-Newtonian flow was explored by Hayat et al. [5]. Effects of variable viscosity in a third grade fluid with porous medium: An analytic solution was discussed by Ellahi and Afzal [6]. When the forced and free convection differences are of harmonious order phenomena mixed convection occurs. It has vital appearance in atmospheric boundary layer flows, heat exchangers, solar collectors, nuclear reactors and in electronic equipment's. These physical processes occurs in the situation where the impacts of buoyancy forces in forced convection or the influences of forced flow in natural convection become much more dominant. The interaction of forced and natural convection is particularly noticeable physically where the forced convection flow has low velocity or moderate and large temperature differences. In the concerned article, a rotating vertical cone is positioned in a non-Newtonian nanofluid with the axis of the cone being in line with the external flow is inspected. Mixed convectional flow with heat and mass transfer problems over cones are widely finds its application in automobile and chemical industries. Some of the applications are design of canisters for nuclear waste disposal, nuclear reactor cooling system, etc. It is found that the unsteady mixed convective flows do not particularly gives similarity solution and for the few years later, several flow problems have been studied, where the non-similarity solutions are discussed. The velocities at edge of boundary layer, the body curvature, the surface mass transfer are responsible for unsteadiness and non-similarity in such kind of fluid flows. Hering and Grosh [7] have obtained a number of similarity solutions for cones with prescribed wall temperature being a power function of the distance from the apex along the generator. In recent studies, Anilkumar and Roy [8] obtained the self-similar solutions of unsteady mixed convection flow from a rotating cone in a rotating fluid. They found that the self-similar solutions are only possible, if the angular velocity at the edge and the angular velocity at the wall of cone vary inversely as a linear function of time. Alamgir [9] presented the overall heat transfer in laminar natural convection flow from vertical cones by using the integral method. The steady free convection boundary layer over a vertical cone embedded in a porous medium filled with a non-Newtonian fluid with an exponential decaying internal heat generation is

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studied by Rashidi and Rastegari [10]. In many cases, the flow can be unsteady due to the time dependent free stream velocity and there are several transport processes with surface mass transfer (suction/injection) where the buoyancy forces arise from the thermal and mass diffusion caused by the temperature and concentration gradients. Therefore, as a step towards the consequent development of investigation on combined convection flows, it is very important and useful to study the unsteady mixed convection boundary layer flow over a vertical cone with the thermal and mass diffusion, when the free stream velocity varies arbitrarily with time [11–13].

The study of convective transport of nanofluids is of superb importance due to its feature to increase the thermal conductivity of fluid as associated to base fluid. The term nanofluid is associated to such type of fluids where the suspension of nano-scale particles and the base fluid is being merged. Choi [14] was the first who used this concept. He exposed that by addition of a small amount of nanoparticles into conventional heat transfer liquids enhanced the thermal conductivity of the fluid approximately two times. A recent application of nanofluid flow is nano-drug delivery [15]. Suspension of metal nanoparticles is also being developed for other purposes, such as medical applications including cancer therapy. Also the nanofluids are regularly used as coolants, lubricants and micro-channel heat sinks. Typically nanofluids consist of metals, oxides or carbon nanotubes. Buongiorno [16] introduced seven slip mechanisms between nanoparticles and the base fluid. He revealed that the Brownian motion and thermophoresis have noteworthy effects in the laminar forced convection of nanofluids. Based on such observations, he established non-homogeneous two-component equations in nanofluids. Thermal Performance of Ethylene Glycol Based Nanofluids in an Electronic Heat Sink was analyzed by Selvakumar and Suresh. [17]. Further Akbar et al. [18] investigated interaction of nano particles for the peristaltic flow in an asymmetric channel with the induced magnetic field. In real situations in nanofluids, the base fluid do not fulfill the properties of Newtonian fluids, hereafter it is more reasonable to consider them as viscoelastic fluids. In the present paper, the base fluid is taken as third grade fluid. Rahmani et al. [19] deliberated the study of thermal and fluid effects of non-Newtonian water-based nanofluids on the free convection flow between two vertical planes. Some experimental and theoretical works related to nanofluids are given in references [14–23].

The core determination of the present work is to analyze the effects of Brownian motion and thermophoresis on mixed convection flow of a third grade fluid on a rotating vertical cone. The nonlinear partial differential equations of third grade nanofluid are initially reduce to system of nonlinear ordinary differential equations with a set of similarity transformations and the solutions are carried out by using homotopy analysis method (HAM) [24–30]. The effects of physical parameters on velocities, surface friction coefficients, temperature and nanoparticle volume fraction are calculated and discussed graphically and numerical tables. Also the results are recovered for the case where nanoparticles are not present.

2. Mathematical analysis

The effects of thermophoresis and Brownian motion on the unsteady mixed convection flow on a rotating cone in a rotating third grade fluid with time dependent angular velocity has been investigated. The flow is assumed to be incompressible, axisymmetric and non-dissipative. The rotation of the cone and the fluid with the axis of cone either in same or in inverse direction causes unsteadiness in the fluid flow. Rectangular curvilinear fixed coordinate system is used to elaborate the geometry of the flow in which x, y and z and are taken along tangential, azimuthal direction and normal directions respectively. Here u, v and w are the components of velocity in x, y and z directions, respectively (see Fig. 1).

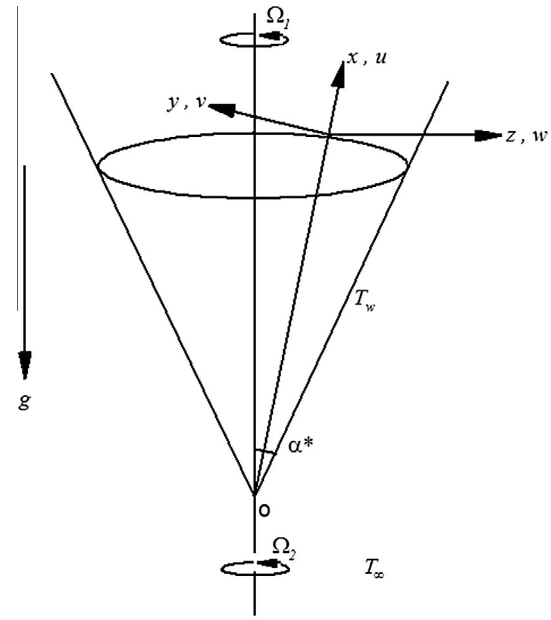

Fig. 1. Geometry of the problem.

The temperature T_w and concentration C_w at the wall are supposed to be linear functions of distance x. The differences in temperature and concentration fields produce the buoyancy forces in the flow. With the help of Boussinesq approximations, the boundary layer flow, temperature and concentration equations for third grade nanofluid are set as

$$\frac{\partial(xu)}{\partial x}+\frac{\partial(xw)}{\partial z}=0, \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-\frac{v^2}{x}=&-\frac{v_e^2}{x}+\upsilon\frac{\partial^2 u}{\partial z^2}\\&+\frac{\alpha_1}{\rho}\left\{\frac{\partial^3 u}{\partial z^2\partial t}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x\partial z}\frac{\partial v}{\partial z}+\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial z^2}\right\}\\&+\frac{\alpha_2}{\rho}\left\{\frac{1}{x}\left(\frac{\partial u}{\partial z}\right)^2+\frac{\partial^2 u}{\partial x\partial z}\frac{\partial u}{\partial z}-\frac{1}{x}\left(\frac{\partial v}{\partial z}\right)^2+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\right\}\\&+\frac{(\alpha_1+\alpha_2)}{\rho}\left\{3\frac{\partial^2 u}{\partial x\partial z}\frac{\partial u}{\partial z}+2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x\partial z}\frac{\partial v}{\partial z}+\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial z^2}-\frac{v}{x}\frac{\partial^2 v}{\partial z^2}\right.\\&\left.-\frac{1}{x}\left(\frac{\partial v}{\partial z}\right)^2+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\right\}+\frac{\beta_3}{\rho}\left[2\frac{\partial^2 u}{\partial z^2}\left\{3\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right.\right.\\&\left.\left.+4\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial z^2}\right\}\right]+g\xi cos\alpha^*(T_0-T_\infty)+g\xi^* cos\alpha^*(C-C_\infty),\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}+\frac{uv}{x}=&\frac{\partial v_e}{\partial t}+\upsilon\frac{\partial^2 v}{\partial z^2}+\frac{\alpha_1}{\rho}\left\{\frac{\partial^3 v}{\partial z^2\partial t}-\frac{v}{x}\frac{\partial^2 u}{\partial z^2}+\frac{u}{x}\frac{\partial^2 v}{\partial z^2}\right\}\\&+\frac{\alpha_2}{\rho}\left\{\frac{\partial u}{\partial z}\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\right\}+\frac{(\alpha_1+\alpha_2)}{\rho}\\&\times\left\{\frac{\partial^2 v}{\partial x\partial z}\frac{\partial u}{\partial z}+\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial z^2}-\frac{v}{x}\frac{\partial^2 u}{\partial z^2}+\frac{2u}{x}\frac{\partial^2 v}{\partial z^2}+\frac{1}{x}\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}+\frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2}+\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\right\}\\&+\frac{\beta_3}{\rho}\left[2\frac{\partial^2 v}{\partial z^2}\left\{\left(\frac{\partial u}{\partial z}\right)^2+3\left(\frac{\partial v}{\partial z}\right)^2+4\frac{\partial u}{\partial z}\frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial z^2}\right\}\right],\end{aligned} \tag{3}$$

$$(\rho c)_f\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}\right)=\kappa\frac{\partial^2 T}{\partial z^2}+(\rho c)_p\left\{D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right\}, \tag{4}$$