International Journal of Heat and Mass Transfer 76 (2014) 99–104

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Efficiency analysis of heat exchangers and heat exchanger networks

Ahmad Fakheri *

Department of Mechanical Engineering, Bradley University, Peoria, IL 61625, United States

A R T I C L E I N F O

Article history:
Received 23 February 2014
Received in revised form 10 April 2014
Accepted 10 April 2014

Keywords:
Heat exchangers
Efficiency
Heat exchanger efficiency
LMTD
Effectiveness-NTU
Heat exchanger networks

A B S T R A C T

The concept of heat exchanger efficiency eliminates the need for charts, or complicated performance expressions, providing a convenient approach for solving heat exchanger rating, and sizing problems, as well as network of heat exchangers. The efficiency of all heat exchangers is determined from a single algebraic expression. This paper is comprehensive, and streamlined presentation of the approach. It also provides a new expression for solving sizing problems, provides a closed form expression for determining the minimum number of heat exchangers needed when one cannot meet the design specifications, presents a new methodology for analyzing network of heat exchangers connected in series, and provides closed form expressions for determining the size and the rate of heat transfer in individual heat exchangers of a network.

1. Introduction

Consider a heat exchanger where the hot fluid with capacity C_h enters at T_1 and leaves at T_2. The cold fluid with capacity C_c enters at t_1 and exits at t_2. The heat exchanger area is A, and the overall average heat transfer coefficient is U, and is assumed to be a constant, independent of temperature. Traditionally, two different methods have been used for analyzing heat exchangers: the Log Mean Temperature Difference (LMTD); and Effectiveness NTU-method (ε-NTU) [1,2]. The LMTD method is generally used for solving heat exchanger problems where the inlet and the exit temperatures are known and the size of the heat exchanger is to be determined (sizing problems). The reverse problem is called the rating problem where the size of the heat exchanger and the inlet temperatures are known and the heat transfer rate and the fluid exit temperatures are sought. The rating problem is typically analyzed using ε-NTU approach. Fakheri [3] recently proposed a third method for analyzing heat exchangers by defining the concept of heat exchanger efficiency.

Heat exchanger efficiency can be used to conveniently analyze different heat exchanger design problems, including the network of heat exchangers without the need for charts, or complicated performance expressions. Efficiency provides a clear and intuitive measure of how well a system is performing by showing how close it comes to the best that it can be and if further improvements are feasible and justified. The heat exchanger efficiency is defined as the ratio of the actual rate of heat transfer in the heat exchanger, q, and the optimal rate of heat transfer, q_{opt},

$$\eta = \frac{q}{q_{opt}} = \frac{q}{U\,A(\overline{T} - \overline{t})} \tag{1}$$

The optimum (maximum) rate of the heat transfer is the product of *UA* of the heat exchanger under consideration and the Arithmetic Mean Temperature Difference (AMTD) in the heat exchanger

$$AMTD = \overline{T} - \overline{t} = \frac{T_1 + T_2}{2} - \frac{t_1 + t_2}{2} \tag{2}$$

which is the difference between the average temperatures of hot and cold fluids. The optimum heat transfer rate takes place in a balanced counter flow heat exchanger [4]. The rate of heat transfer in any heat exchanger for the same *UA* and *AMTD* is always less than the optimum value of the heat transfer rate ($\eta \leqslant 1$).

The heat transfer in a heat exchanger can be calculated from

$$q = \eta NTUC_{\min}(\overline{T} - \overline{t}) \tag{3}$$

The amount of heat transfer is also equal to

$$q = C_{\min}\Delta T_{\max} = C_{\max}\Delta T_{\min} \tag{4}$$

Note that some authors use $\Delta T_{\min}$ and $\Delta T_{\max}$ to refer to the temperature changes of the fluids with smaller and larger capacities, respectively, which is opposite of how it is used here.

The efficiency of all heat exchangers can be expressed as [2]

$$\eta = \frac{\tanh[Fa]}{Fa} \tag{5}$$

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* Tel.: +1 3096772719.
E-mail address: ahmad@bradley.edu

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.04.027
0017-9310/

Nomenclature

A	surface area, m^2
$AMTD$	Arithmetic Mean Temperature Difference; $AMTD = (\frac{T_1+T_2}{2} - \frac{t_1+t_2}{2}) = (\overline{T} - \bar{t})$
C_c	heat capacity of the cold fluid $C_c = (\dot{m}C_p)_c$
C_h	heat capacity of the hot fluid $C_h = (\dot{m}C_p)_h$
$C_{\min}$	$=\min[C_h, C_c]$
$C_{\max}$	$=\max[C_h, C_c]$
C_r	capacity ratio $C_r = \frac{C_{\min}}{C_{\max}}$
Fa	Fin analogy number Eq. (6)
m	Fa coefficient given in Table 1
n	Fa coefficient given in Table 1
N	number of shells
NTU	number of transfer units $NTU = \frac{UA}{C_{\min}}$
q	rate of heat transfer; $q = UA\eta(\overline{T} - \bar{t})$
q_{opt}	optimum heat transfer rate $q_{opt} = UA(\overline{T} - \bar{t})$
T_1	hot fluid inlet temperature
T_2	hot fluid exit temperature
t_1	cold fluid inlet temperature
t_2	cold fluid exit temperature
$\overline{T}$	average temperature of the hot fluid $\overline{T} = \frac{T_1+T_2}{2}$
$\bar{t}$	average temperature of the cold fluid $\bar{t} = \frac{t_1+t_2}{2}$
ΔT	$=T_1 - T_2$
Δt	$=t_2 - t_1$
$\Delta T_{\max}$	$=\max[\Delta T, \Delta t]$
$\Delta T_{\min}$	$=\min[\Delta T, \Delta t]$
U	overall heat transfer coefficient, W/m^2 K
η	heat exchanger efficiency

which is the same form as the efficiency of a constant area insulated tip fin. Fin Analogy number, Fa, is a nondimensional group that characterizes the performance of different heat exchangers and can be written as

$$Fa = \frac{NTU}{2}(1 + mC_r^n)^{\frac{1}{n}} \tag{6}$$

and therefore the efficiency of all heat exchangers can be determined from

$$\eta = \frac{\tanh\left[\frac{NTU}{2}(1 + mC_r^n)^{\frac{1}{n}}\right]}{\frac{NTU}{2}(1 + mC_r^n)^{\frac{1}{n}}} \tag{7}$$

the values of m and n are given in Table 1 for different heat exchanger types. The values for cross flow heat exchangers are approximate and have been obtained by using regression analysis [5].

The LMTD correction factor, the heat exchanger effectiveness, and heat exchanger efficiency are all derived from the same basic set of equations and therefore can be related to each other. For example, efficiency is related to heat exchanger effectiveness Fakheri [6] through

$$\eta = \frac{1}{NTU}\frac{1}{\frac{1}{\varepsilon} - \frac{(1 + C_r)}{2}} \tag{8}$$

Eq. (8) is a general expression that can be used to determine the efficiency of any heat exchanger, when its effectiveness is known.

2. Rating problem

If the size of the heat exchanger, the fluid capacities, and the inlet temperatures are known, then the objective is to determine the exit temperatures. This type of problem is called heat exchanger rating problem. In Eq. (3) all the terms on the right hand side are known, except the mean temperature difference. Rearranging Eq. (2) and substituting for the change in temperature of hot and cold fluids from Eq. (6) results in

$$\begin{aligned}\overline{T} - \bar{t} &= \frac{1}{2}(2T_1 - (T_1 - T_2) - 2t_1 - (t_2 - t_1)) \\ &= T_1 - t_1 - \frac{q}{2}\left(\frac{1}{C_h} + \frac{1}{C_c}\right) = T_1 - t_1 - \frac{q}{2C_{\min}}(1 + C_r)\end{aligned} \tag{9}$$

Table 1
Fin analogy number parameters for different heat exchangers.

HX type	m	n
Counter flow	−1	1
Parallel	1	1
Single stream ($C_r = 0$)	1	1
Single shell and tube	1.	2
Cross flow, $C_{\max}$ unmixed, $C_{\min}$ mixed	1.2	4.4
Cross flow, $C_{\max}$ mixed, $C_{\min}$ unmixed	1.35	4.02
Cross flow, both mixed	1.2	2
Cross flow, both unmixed	−0.1	0.37

Substituting for q from Eq. (3) and solving for the mean temperature difference results in

$$\overline{T} - \bar{t} = \frac{T_1 - t_1}{1 + NTU\eta\frac{(1 + C_r)}{2}} \tag{10}$$

Everything on the right hand side of the Eq. (10) is known, allowing the determination of the mean temperature difference, which can then be substituted in Eq. (3) to determine heat transfer, and subsequently the exit temperatures of hot and cold fluids. Alternatively a direct expression for determining heat transfer in the heat exchanger can be determined by substitution from Eq. (10) into Eq. (3)

$$q = C_{\min}\frac{T_1 - t_1}{\frac{1}{\eta NTU} + \frac{(1 + C_r)}{2}} \tag{11}$$

Note that the term in the denominator of Eq. (11) is the inverse of the heat exchanger effectiveness.

3. Sizing problem

When the fluid capacities and their inlet and exit temperatures are known or can be determined, then determining the size of the heat exchanger becomes the objective. This type of problem is called heat exchanger sizing problem. The heat transfer and the temperature difference are known, then from Eq. (3)

$$NTU\,\eta = \frac{q}{C_{\min}(\overline{T} - \bar{t})} \tag{12}$$

From Eq. (5)

$$Fa = \tanh^{-1}\left[\eta NTU\frac{Fa}{NTU}\right] \tag{13}$$

Substituting for ηNTU from Eq. (12) and for Fa from Eq. (6) results in

$$Fa = \tanh^{-1}\left[\frac{q}{C_{\min}(\overline{T} - \bar{t})}\frac{(1 + mC_r^n)^{\frac{1}{n}}}{2}\right] \tag{14}$$

Substituting for q and capacity ratio from Eq. (6) into Eq. (14)

$$Fa = \tanh^{-1}\left[\frac{\Delta T_{\max}}{(\overline{T} - \bar{t})}\frac{\left(1 + m\left(\frac{\Delta T_{\min}}{\Delta T_{\max}}\right)^n\right)^{\frac{1}{n}}}{2}\right] \tag{15}$$