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Effect of memory accumulation in three-scale fractured-porous media

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ABSTRACT

We consider the flow of low compressible liquid described by the diffusion equation, which is also identical to the conductive heat transport, in a triple porosity medium that consists of two hierarchical connected networks of thin fractures and isolated low permeable blocks between the fractures. We reveal a unique parameterization of fracture thicknesses and permeabilities that ensures the contribution of all three medium subdomains into the macroscopic behavior. Such a parameterization corresponds to a non-selfsimilar medium.

In such a medium the main flow occurs through the large-scale fractures, the small fractures/fissures play the role of fluid sources for large fractures, while the porous blocks play the role of fluid sources for small fractures.

The delay in flow between different sub-domains leads to the appearance of memory in the macroscopic model described by the integro-differential operators. The double delay between three scales leads to the effect of memory accumulation, in such a way that the memory kernel of the integral operators is it-self the solution of an integro-differential cell problem. For thin fractures, the memory kernels have a specific structure that corresponds to Abel's type.

The macroscopic model is obtained by means of the asymptotic two-scale homogenization applied sequentially twice. These two sequential steps are non-symmetrical. At the first scale one deals with the problem of flow in continuous network of thin fractures/fissures surrounded by blocks, but only the boundary layer in each block is perturbed. At the second step we homogenize the flow in the medium that consists of large connected fractures and averaged blocks. Each averaged block contains a lot of small fractures and small porous blocks. At this step, in contrast, the appearance of a boundary layer inside the averaged blocks is prohibited.

This model is used to calculate flow around a producing well in an oil reservoir, in comparison with the double porosity medium. The qualitative difference is revealed.

1. Introduction

The typical structure of natural porous rocks represents a hierarchical system of several fracture networks, included into one another, having different permeabilities and apertures. Respectively the porous blocks situated between the fractures and isolated from each other also form a hierarchical system. Thus, one deals with hierarchical, or multiscale media. At each scale the characteristic heterogeneity length by the order of ε ($\varepsilon \ll 1$) is small with respect to the macroscale, which gives the possibility to neglect the details of flow at all small scales and to effectively describe the process in terms of macroscopic models, homogenized over all the heterogeneities.

The vuggy porous media with highly permeable or even open fractures where the fractures are assumed to be void channels and flow can be described by the Stokes–Brinkman equations has been studied in [6,13]. In the case of porous fractures, flow is described by Darcy's law.

Frequently *double-porosity* model, proposed in [20], is used to study fractured-porous media, which is not too exact. *Double-porosity* model is proposed for the so-called *suger cube* geometry, where blocks are not interconnected with each other and are surrounded by fractures. Several studies have considered the effect of different geometries and fracture-block arrangements on the overall flow. In [19], they have considered the case where global flow in the matrix blocks plays an important role as the matrix blocks are connected to neighboring blocks. In [14], two arrangement of a

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quadruple fractured medium was studied: when the small fractures are interconnected only with medium fractures and the case that small fractures are connected to medium and large fractures. They have shown that the difference between the two cases are not negligible. Semi-analytical solutions for continuously and discretely fractured reservoirs has been provided in [7,12]. They have shown that the *double-porosity* media approximation cannot be simply applied to any fractured media and the role of geometrical parameters of the domain is not negligible which would change the overall behavior of the system. They have shown that the thin fractures may not participate in general flow system. The high permeability of fractures ensures their dominant role in the transport. However if the fracture is very thin, then it becomes even less penetrable to fluid than the low permeable blocks.

In this paper we analyze fractured-porous media, which consist of tight porous blocks crossed by two different networks of porous fractures, connected between them, but having different thicknesses, lengths and permeabilities. Each network is characterized by a small distance between two neighboring fractures, of order of ε. The second small parameter that appears is the order of ratio between the permeabilities of two successive media, ω. The permeability of large fractures is expected to be higher than that of small fractures, and the latter is much higher than the permeability of tight blocks. The fractures are assumed to be very thin. This is their principal difference from *double-porosity media* in which the fracture aperture is of the same order as the size of a porous block. A third small parameter is h the fracture aperture which changes the structure of asymptotic expansions and, consequently, the macroscale behavior. The regimes of flow depend significantly on the ratio between the three small parameters: ε, ω and h [2,14,19].

It is possible to find such a ratio between the parameters ε, ω and h that provides the most general types of macroscopic model. We will call it the canonical model. For instance, in *double-porosity* media, the canonical model corresponds to $\omega = \varepsilon^2$ [5,8,9,15]. In this case the homogenization[11] of the diffusion equation leads to the appearance of the memory described by an integro-differential operators responsible for the mass exchange between blocks and fractures. The memory appears due to a significant delay that exists between the flow through a fracture and a block.

Extension of *double-porosity* medium to multiscale porous medium which is ε^2 at each scale was analyzed in [10,18,17]. In [17], the closed macroscale model was obtained for any number of scales, including the limit model for infinite hierarchy. Such a limit model represents two integro-differential equations whose kernel represents the memory accumulation effects.

The canonical model for two-scale media with thin fractures was first obtained by [2,4]. They have shown that the most general behavior, including the memory appearance, corresponds to the ratio $\omega/h \sim \varepsilon^2$. This is the same as $\omega \sim \varepsilon^{2+\alpha}$ and $h \sim \varepsilon^{\alpha}$, with $\alpha > 0$. It was also shown that during the flow process, practically the overall volume of each porous block remains unperturbed, and only a thin boundary layer in each block is active just in contact with the fracture. The macroscopic flow is thus determined by thin fractures and thin boundary layer in blocks. The boundary layer changes qualitatively the structure of Green's functions of the operators describing flow in a block, and the respective structure of the memory kernels in the macroscopic equations.

For three-scale media with two types of thin fractures, the canonical model remains an open problem. The main problem of three-scale medium consists of the memory appearance and memory accumulation at each new step of homogenization. Due to this the flow model changes its type and form at each step. The form of the final model is unknown a priori and cannot be predicted. The triple-porosity medium with thin fractures was obtained in [3], where the case of geometrically self-similar medium was analyzed, which implies the same ratio between parameters ε, ω and h at each scale. This case does not provide the most general, canonical model, as it corresponds to a particular situation when the contribution of the third scale in the macroscopic flow is very low.

In the present paper we construct the canonical model for three-scale medium with thin fractures, which proves the most general behavior. We apply the iterative procedure based on the asymptotic two-scale homogenization method. First of all the couple of porous blocks and the small-scale fracture network are homogenized. Secondly the obtained homogenized medium in couple with the large-scale fractures is homogenized, which yields the macroscopic flow equations. The effective cumulated memory of the system is determined through the solution of an integro-differential equation. We compare the results provided by the two-scale and a three-scale model, which are in good correspondence with physical meaning. In particular, during the oil recovery from reservoir the pressure in three-scale medium should be higher than that in two-scale medium (due to a double system of sources which aliments large fractures).

2. Physical and mathematical formulation

2.1. Medium geometry

Let us consider a three-scale fractured medium consisting of the system of homogeneous low permeable, non-intersecting porous cubes Ω^1 and the network of thin porous fractures Ω^2, Ω^3 as it is shown in Fig. 1. In general case the cubes cannot be necessarily true cubes, so we will call them "the blocks". We introduce the medium $\widetilde{\Omega}^2$ as the superposition of non-intersecting blocks Ω^1 and the fracture Ω^2. The ratio between the linear sizes of the periods of media $\widetilde{\Omega}^2$ and Ω^1 is a constant small value of order of $\varepsilon \ll 1$.

Any fracture domain Ω^3 is self-connected within the corresponding period of the medium $\widetilde{\Omega}^2$. Moreover it is directly connected to the fractures of the smaller scale Ω^2, but has no direct connection with the blocks Ω^1. Physically these fractures are highly permeable porous channels. The periods of networks Ω^3 and Ω^2 are $\varepsilon \ll 1$ and ε^2, respectively. To characterize the absolute sizes of each medium, we introduce the main coordinate system $x = x^{(3)}$ associated with the largest network of fractures, Ω^3. The notation $x^{(3)}$ means a vector in $R^3 : x^{(3)} = \left\{x_1^{(3)}, x_2^{(3)}, x_3^{(3)}\right\}$.

Along with the global system of coordinates x, we introduce two local coordinate frames $y^{(2)}$ and $y^{(1)}$, Fig. 1, which correspond to two smaller scales which are defined as:

$$y^{(2)} = \frac{x}{\varepsilon}, \quad y^{(1)} = \frac{x}{\varepsilon^2} \tag{1}$$

They vary within the elementary cells $Y^3 \cup \widetilde{Y}^2$ and $Y^2 \cup Y^1$ shown in Fig. 2(a) and (b) and are defined as:

$$Y^3 \cup \widetilde{Y}^2 = \left\{-\frac{1}{2} < y_i^{(2)} < \frac{1}{2}\right\}, \quad Y^2 \cup Y^1 = \left\{-\frac{1}{2} < y_i^{(1)} < \frac{1}{2}\right\},$$
$$i = 1, 2, 3$$

so that $Y^3 \cup \widetilde{Y}^2$ is the image $x \mapsto y^{(2)}$ of the unit period of the domain $\widetilde{\Omega}^3$, while $Y^2 \cup Y^1$ is the image $x \mapsto y^{(1)}$ of the unit period of the domain $\widetilde{\Omega}^2$. The linear size of any elementary cell in local coordinates is equal to 1, as shown in Fig. 2.

The half-thickness of a fracture $\Omega^i, h^{(i)}$ is much smaller than the period of the same fracture network, so that: where $h^{(3)} \ll \varepsilon, \quad h^{(2)} \ll \varepsilon^2, h^{(3)}$ and $h^{(2)}$ are measured in the global coordinate system x shown in Fig. 1.

2.2. Flow equations

The single-phase flow of a slightly compressible liquid in porous medium can be described by the linear diffusion equation: