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Numerical modeling of turbulent flow through isotropic porous media

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ABSTRACT

Turbulent flow through porous media is of considerable theoretical and practical interest in various engineering disciplines such as chemical, mechanical, nuclear, geological, environmental, petroleum, etc. In the present work, a numerical study of turbulent flow through porous media was carried out. The porous media was considered to be made of a spatially periodic array of infinite square cylinders. The low *Re*–k–ε–*Lam*–*Bremhorst* (*LB*) turbulent model was employed for the description of the turbulent flow in the porous media. The transport equations were solved only through the flow domains having different porosities. The porosity of the medium (ϕ) and the Reynolds number (Re_D), were taken in the range of $0.3 \leqslant \phi \leqslant 0.84$, and $100 \leqslant Re_D \leqslant 40{,}000$, respectively. The influence of ϕ and Re_D on normalized turbulent kinetic energy, $\langle k \rangle^f$ and dissipation rate, $\langle \varepsilon \rangle^f$ was investigated and the simulated data were compared with two different turbulent flow models, namely, large eddy simulation (LES), and v2f model. The predictions of turbulent parameters using low *Re* k–ε–*LB* turbulent model were found to be in good agreement with the LES than that with v2f model. The macroscopic pressure variation across the flow domain was also investigated and the predictions of the pressure gradient were found to be in good agreement with the Forchheimer-extended Darcy law. At low Re_D ($Re_D = 100$) the low *Re*–k–ε–*LB* predictions were deviating from the DNS, LES and v2f predictions. This means that the proposed low *Re*–k–ε–*LB* model will hold valid only for flows having $Re_D > 300$. A correlation for macroscopic pressure gradient as a function of porosity is also proposed for high Re_D.

1. Introduction

According to Darcy [1], a porous medium is "system ensembles of a solid matrix with its void space filled with fluids". The voids are generally interconnected so as to allow fluid to flow through the porous medium. The solid matrix may either be rigid or deformable, and either consolidated or unconsolidated, with its structure being either ordered or disordered [2]. The analysis of fluid flowing through porous media has applications in a number of industries such as chemical, mechanical, nuclear, petroleum, geological, environmental and specially in engineering systems based on fluidized bed combustion, enhanced oil reservoir recovery, combustion in an inert porous matrix, underground percolation of chemical waste and chemical catalytic reactors, etc. [3]. Reservoir engineering deals with the flow of fluids and fluid mixtures through porous media since the advent of oil production, particularly in oil recovery processes.

Various flow regimes are encountered during the fluid transport in porous media, based on the pore Reynolds number, Re_D, [4]: (a) Darcy or creeping flow regime ($Re_D < 1$); (b) Forchheimer flow regime ($1 < Re_D < 150$); (c) post-Forchheimer flow regime (unsteady laminar flow, $150 < Re_D < 300$); and (d) fully turbulent flow regime ($Re_D > 300$). Such flows can be analyzed by modeling the topology of the porous medium and resolving the microscopic flow equation in the fluid phase. The range of pore-size is large, and can vary over 8-order magnitude spanning Angstrom (ultra-micro-pores) to centimeters (pebbles) or even larger [5]. The conditions encountered in different applications are broad enough to cover a large range of Reynolds numbers. Due to lack of geometric information to model each and every pore, such systems are difficult to simulate with full geometric details. Despite the fact that it might be possible to describe some of these systems in an almost exact representation or a meaningful statistical approximation of the geometry, the computational effort required to solve the flow field in such geometries is still formidable [6]. This motivates further research in the development of porous media approximation, representing the system composed of macroscopic homogenous pores with uniform properties.

The early works on fluid flow through porous media were based mainly on semi-empirical laws [7]. Vafai and Tien [8] derived the general equations for fluid flow through porous media using the

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Nomenclature

C_1, C_2, $C_\mu\sigma_k$, σ_ε turbulence model dimensionless constants
f_1, f_2 and f_μ damping functions
D diameter of square cylinder (m)
d_{eq} equivalent pore diameter (m) $\left(=\sqrt{\frac{32}{150}\frac{\phi}{(1-\phi)}}\cdot D\right)$
H center to center distance between two cylinders (m)
K porous media permeability (m^2) $\left(=\frac{\phi^3}{144(1-\phi)^2}\cdot D^2\right)$
k turbulent kinetic energy (m^2/s^2)
f friction factor $\left(=\frac{(-d\langle p\rangle^f/dx)}{(\rho u_D^2/2d_{eq})}\right)$
$\langle k\rangle^f$ intrinsic volume average turbulent kinetic energy (m^2/s^2)
Re_D Reynolds number based on pore diameter D ($=u_D D/\nu$)
$Re_{D_{eq}}$ equivalent Reynolds number based on Darcy velocity $\left(=\frac{u_D d_{eq}}{\nu}\right)$
Re_y turbulent Reynolds number based on y $\left(=y\sqrt{k}/\nu\right)$
Re_t turbulent Reynolds number ($=k/\nu\varepsilon$)
u velocity vector (m/s)
$\bar{u}$ time average velocity vector (m/s)
u' fluctuating velocity vector (m/s)
u_D Darcy velocity vector (m/s)
V_f fluid volume inside the porous media (m^3)
X, Y horizontal and vertical co-ordinate

Greek symbols
ε dissipation rate of k (m^3/s^2)
$\langle\varepsilon\rangle^f$ intrinsic volume average turbulent kinetic energy dissipation rate (m^3/s^2)
$\langle\psi\rangle^v$ volume average fluid property
$\langle\psi\rangle^f$ intrinsic average fluid property
μ dynamic viscosity (kg/m s)
ν kinematic viscosity (m^2/s)
μ_t turbulent viscosity (kg/m s)
ρ Fluid density (kg/m^3)
ϕ porosity
ΔP pressure drop across the flow domain

volume-averaging technique. This technique is a rigorous mathematical procedure for deriving the governing mass, momentum and energy equations in porous media [9,10]. Several researchers derived the macroscopic turbulence model using the similar methodology [11–13,4,14–17]. Masuoka and Takatsu [11] derived a zero-equation turbulence model using the local volume-averaging technique. They modeled the effective eddy diffusivity as the algebraic sum of the eddy diffusivities estimated from two types of vortices – the pseudo vortex and the interstitial vortex. Antohe and Lage [12] derived a two-equation macroscopic turbulence model applying the time averaging operator to the extended Darcy–Forchheimer model. Applying the volume-averaging theory to the microscopic transport equations of turbulent kinetic energy and its dissipation rate, Nakayama and Kuwahara [13], and Pedras and de Lemos [4], established a macroscopic two-equation turbulence model. They obtained a new set of equations for the transport of the volume averaged turbulence kinetic energy and its dissipation rate. Lee and Howell [14] proposed the k–ε model for flow through porous media with high porosity and used pure fluid with eddy viscosity for porous media. Travkin et al. [15] developed the turbulence model for highly porous media along with statistical and numerical methodologies. Macdonald et al. [16] and Dybbs and Edwards [17] performed experiments on fluid flow in porous media, and observed highly unsteady chaotic flow within the pores and constant friction coefficient for higher Reynolds number (at $Re_D > 300$, based on the average pore dimension, and $Re > 1000$ based on the average pore velocity).

A variety of natural and engineering systems can be characterized as porous structure through which a fluid flows in turbulent regime. In petroleum extraction, the flow accelerates toward the pumping well while crossing the regions of variable porosity. During the fluid transport, the flow regime gradually shifts from laminar to transition and eventually to turbulent regime. This affects the overall pressure drop and well performance. The fluidized-bed combustors and chemical catalytic reactors also experience pressure loss variation due to changes in the flow regime inside the pores. Also, the turbulence within the porous zone plays a significant role in the thermal processes. The wind transport of pollutants through forests, crop lands and ventilated structures is also modeled as flow through porous media. The geometric structures of the above areas and the wind speed induce turbulence generation [12]. Turbulent boundary layers over forests and vegetation are examples of important environmental flows which can benefit from appropriate mathematical modeling. In all cases, better understanding of turbulence through adequate modeling can, more realistically, simulate real-world environmental/natural and engineering flows.

A number of turbulence models such as, Reynolds average Navier–Stokes (RANS) and LES based simulation models are available for studying the turbulence phenomena. Besides, the Navier–Stokes equations are numerically solved by using direct numerical simulation (DNS) in computational fluid dynamics encompassing the entire flow domain. Direct numerical simulation (DNS) is a simulation in computational fluid dynamics, wherein the Navier–Stokes equations are numerically solved all possible degrees of freedom appearing in the flow field without any turbulence model and approximating assumptions. The DNS offers insight into the mechanism of drag reduction [18], although its application in engineering flows is generally restricted to $Re < 10{,}000$ [19]. As the Re increases, the number of grid points required for the simulation increases drastically according to the relation $N_{G,DNS} \geq Re_t^{2.24}$ [20,21]. Thus, the memory and the computational speed of the computing system restrict the range of Re for the application of DNS. It may, however, be noted that the DNS has been used even at supersonic base flow at $Re = 100{,}000$ [22]. The LES model uses filtered three-dimensional unsteady Navier–Stokes equations separating the large scale eddies from the small scale ones. Thus, the main flow structure is resolved directly, and the small scale eddies are modeled by a subgrid scale model [23]. However, the DNS and LES models require large memory and CPU time. Due to these limitations, the RANS model is preferred for engineering problems. The low Re k–ε model provides stable converged solution readily as compared to DNS and LES. This model also requires relatively low computational effort [12,24].

The standard Re k–ε model (Launder and Sharma) is applicable for high-Reynolds number flows and employs wall functions [25]. Near-wall treatment is very important in wall-bounded turbulent flows [26]. Since high velocity gradients exist near the wall, the standard k–ε model is modified to extend its applicability to the low-Reynolds number (Re) flows, which allows calculations right up to the wall. Although the low Re k–ε model is well-structured mathematically, the strong nonlinearities may interact with