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Inclusion of the frequency effect in the lumped parameters transmission line model: State space formulation

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ABSTRACT

The objective of this paper is to include the frequency dependence of the longitudinal parameters in the lumped parameters line model. The distributed nature of the transmission line was approximated by a cascade of π circuits and the frequency effect was approximated by a rational function which was synthesized by an equivalent circuit. Then, the equivalent circuit was inserted in each π circuit of the cascade. After that, the currents and voltages along the line were described in the form of state equations. This way, it was possible to obtain a formation rule of the state matrices lumped parameters model taking into account the frequency dependence. To confirm the validation of the state matrices obtained, the lumped parameters representation of frequency-dependent lines was used to represent a single-phase line and a three-phase line. The simulations were carried out using state space techniques and an electromagnetic transient program (EMTP) (in this case, the cascade was inserted in the EMTP). It is observed that the simulation results obtained with state space representation are in agreement with those results obtained with EMTP.

1. Introduction

Transmission overhead line models used for transient simulations are frequently classified as being a lumped and distributed parameters model.

To obtain a lumped parameters line model, a given line is represented by connecting n short nominal π circuits in cascade. Considering the maximum transient frequency and the line length if a sufficiently high number of sections is chosen the distributed nature of the transmission line can be simulated adequately.

When a lumped parameters line model is adopted, it is very common to use state space techniques to evaluate the currents and voltages along the line. In this way, it is possible for the model to carry out simulations directly in time domain without the explicit use of inverse transforms and it can be easily implemented. These characteristics of the lumped parameters model are the same as those used to simulate electromagnetic transients on lines with nonlinear components, such as corona effects and fault arcs [1–3], or when a detailed voltage and current profile is needed [4].

However, several papers show that the lumped parameters model is developed without taking into account the frequency dependence of the longitudinal parameters of the line [1–6] and this fact limits the accuracy of the lumped parameters model because it is known that models which assume constant parameters cannot adequately simulate the response of the line over the wide range of frequencies that are present in the signals during transient conditions.

Because the frequency dependence is one of the most important aspects in the modeling of transmission lines for electromagnetic transient studies and the lumped parameters line model has several applications [1–6] we decided to improve this model. To improve it, we developed the state matrices of the lumped parameters line representation taking into account the frequency dependence of the longitudinal parameters.

To improve the lumped parameters model, the longitudinal parameters of the line was approximated by a rational function and, after that, this rational function was associated with an equivalent circuit. Then, the longitudinal parameters fitted by the equivalent circuit were inserted in each π circuit. Finally, we wrote the state equations for the currents and voltages along the cascade and we also found a formation rule to evaluate the matrices of the state space representation.

The proposed frequency-dependent model was used to represent a single-phase line and a non-transposed 440 kV three-phase line.

2. Frequency dependence of the longitudinal parameters

It has long been recognized that one of the most important aspects in the modeling of transmission lines for electromagnetic

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transient studies is to account for the frequency dependence of the parameters. In most cases the constant parameter representation produces a magnification of the higher harmonics of the signals and, as a consequence, there is a general distortion of the wave shapes and exaggerated magnitude peaks [7].

The parameters of transmission lines with ground return are highly dependent on the frequency. Formulas to calculate the influence of the ground return were developed by Carson and Pollaczek and these formulas can also be used for power lines. Both seem to give identical results for overhead lines, but Pollaczek's formula is more general inasmuch as it can also be used for underground conductors or pipes [8].

The skin effect, which is related to the inner impedance, results from the electromagnetic field within the conductor. It only belongs to the self-impedance and consists of a frequency-dependent resistance and inductance, which can be calculated, with good accuracy, with formulas based on Bessel functions of conductor geometric parameters, material electric conductivity and frequency, or with simplified formulas depending on frequency range. Due to the skin effect, the resistance increases whereas the inductance decreases [9].

3. Fitting longitudinal parameter of the line

To include the frequency dependence of the longitudinal parameters in the state matrices, initially it is necessary to approximate it by a rational function which can be associated with an equivalent circuit representation [10].

The per unit length (*p.u.l.*) longitudinal impedance $Z(\omega)$ of a transmission line, tabulated taking into account soil and skin effects, is an improper function. In this way, for fitting $Z(\omega)$ it is necessary initially to obtain a modified function $F(\omega)$ as follows [10]:

$$F(\omega) = \frac{Z(\omega) - R_{dc}}{j\omega} \tag{1}$$

where R_{dc} is the asymptotic value of $Z(\omega)$ for $\omega = 0$.

Let us consider a rational function $F(\omega)_{fit}$ written as being:

$$F(\omega)_{fit} = d + \sum_{n=1}^{N} \frac{c_n}{j\omega - a_n} \tag{2}$$

In (2) a_n is a real and negative pole, c_n is a real and positive zero and d is a real and positive residue.

If it is considered that $F(\omega)$ can be approximated by $F(\omega)_{fit}$, we get:

$$F(\omega) \approx F(\omega)_{fit} \tag{3}$$

Substituting (1) and (2) in (3), we get:

$$Z(\omega) \approx R_{dc} + j\omega d + \sum_{i=1}^{m} \frac{j\omega c_i}{j\omega - a_i} \tag{4}$$

The right side in (4) will be denominated $Z(\omega)_{fit}$. This function is an analytical function written as being:

$$Z(\omega)_{fit} = R_{dc} + j\omega d + \sum_{i=1}^{m} \frac{j\omega c_i}{j\omega - a_i} \tag{5}$$

Eq. (5) shows that the tabulated function $Z(\omega)$ can be approximated by a analytical and rational function $Z(\omega)_{fit}$.

Many fitting procedures are available to obtain an approximated rational function for $F(\omega)$ starting from tabulated values from $Z(\omega)$ and in this paper the Vector Fitting algorithm [10,11] will be used. In a general way, the Vector Fitting algorithm is accurate, robust and can be applied to both smooth and resonant responses with high orders and wide frequency bands.

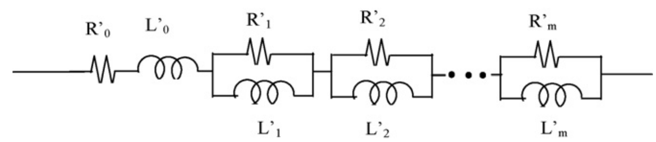

Fig. 1. Equivalent circuit used for fitting the *p.u.l.* longitudinal parameters of a transmission line.

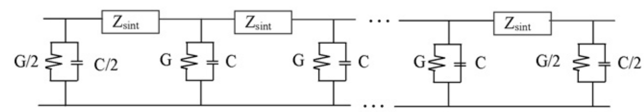

Fig. 2. Single-phase line with length *l* modeled by a cascade with *n* π circuits.

Once the function $Z(\omega)_{fit}$ has been fitted, it is possible to associate it with the equivalent circuit shown in Fig. 1.

In Fig. 1, $R'_0, R'_1, R'_2, \ldots, R'_m$ are resistors and $L'_0, L'_1, L'_2, \ldots, L'_m$ are inductors.

The impedance $Z(\omega)_{sint}$ of the circuit shown in Fig. 1 can be expressed as

$$Z(\omega)_{sint} = R'_0 + j\omega L'_0 + \sum_{i=1}^{m} \frac{j\omega R'_i}{j\omega + (R'_i/L'_i)} \tag{6}$$

From (5) and (6) follows:

$$R'_0 = R_{dc} \tag{7}$$

$$L'_0 = d \tag{8}$$

$$R'_i = c_i \tag{9}$$

$$L'_i = -\frac{c_i}{a_i} \tag{10}$$

Therefore, the *p.u.l.* longitudinal impedance of a transmission line can be approximated by the circuit shown in Fig. 1 and the elements of this circuit are calculated by using a fitting method.

4. Including the soil and skin effects in the lumped model

Let us consider that a single-phase line is approximated by a cascade of *n* π circuits. If soil and skin effects are taken into account, the cascade of π circuits will be a network as shown in Fig. 2.

In Fig. 2, there is in each π circuit a longitudinal block denominated Z_{sint}. This block is the equivalent circuit representation of the rational function used for fitting longitudinal parameters of the line. The content of the block Z_{sint} is shown in Fig. 3.

In Fig. 3, the parameters of each π circuit is calculated as being:

$$R_k = R'_k \times \left(\frac{l}{n}\right) \tag{11}$$

$$L_k = L'_k \times \left(\frac{l}{n}\right) \tag{12}$$

$$C = C' \times \left(\frac{l}{n}\right) \tag{13}$$

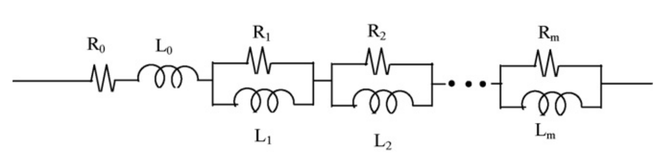

Fig. 3. Frequency effect in the block Z_{sint} of each π circuit.