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Assessment of RANS and improved near-wall modeling for forced convection at low Prandtl numbers based on LES up to $Re_\tau = 2000$

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ABSTRACT

Most promising Generation IV nuclear reactor concepts are based on a liquid metal coolant. However, at low Prandtl (Pr) numbers such as those of liquid metal, classical approaches derived for unity, or close to unity, Pr fail to accurately predict the heat transfer. This paper assesses the RANS modeling of forced turbulent convection at low Pr and in channel flow. Reference results at high Reynolds (Re) number are required to ensure that the Peclet number is sufficiently high. Therefore, new reference results were obtained by performing a wall-resolved Large-Eddy Simulation of turbulent channel flows at a friction Reynolds number $Re_\tau = 2000$ and at $Pr = 0.01$ and 0.025 (this also corresponds to the highest Re Direct Numerical Simulation (DNS) available in the literature for the flow, but without heat transfer). The LES velocity statistics are in very good agreement with those of the DNS and, as validated by the authors in previous publications, the LES approach used here accurately predicts the temperature statistics at low Pr. The LES results are used to assess RANS heat transfer modeling based on the effective turbulent Prandtl number (Pr_t) concept. Among existing Pr_t correlations, the correlation by Kays (1994) [10] is shown to yield the best results. Since it is also shown that the near-wall temperature profile does not follow a log-law, a new "law of the wall for temperature" is here proposed, which does not use any blending function. Its use as a wall-function is also validated in actual RANS simulations.

1. Introduction

Liquid Metal Reactors (LMR) represent a promising technology for achieving the various criteria required to be certified as a GEN-IV concept. For those reactors, two coolants are envisaged: sodium and lead–bismuth eutectic (LBE). The Prandtl number of such fluids is very low, typically of the order of magnitude $Pr \approx 0.01$. At such Prandtl number, the temperature field is much smoother than the velocity field, i.e. the smallest temperature scales are much larger than those of the velocity, and, for moderate Reynolds numbers, the heat transfer in the channel could be essentially molecular while the flow is fully turbulent. Experimental data of heat transfer in liquid metals were mostly obtained in the 60's and 70's and provided global heat transfer correlations for particular geometries, e.g. see the review by Mikityuk [1] for rod bundle data. The numerical simulation has now become an essential design tool and therefore must be accurate in the low Prandtl number regime.

A good overview of issues related to numerical simulation of turbulent heat transfer in liquid metal conditions can be found in the review paper of Grötzbach [2]. Among the most famous works dealing with DNS at low Prandtl number, those of Kawamura et al. [3] and Abe et al. [4] provide interesting results for various Reynolds numbers up to $Re_\tau = 1020$ for Prandtl numbers down to 0.025. For nuclear thermal–hydraulics applications, Tiselj [5] used a spectral code to perform DNS at various Reynolds up to $Re_\tau = 590$ and $Pr = 0.01$. Those studies clearly show that the classical Reynolds analogy is not valid at low Prandtl numbers. However, the similarity between the velocity and temperature fields is the basis of the turbulent Prandtl Pr_t concept and the wall-function approach, and it thus fails to correctly predict the local heat transfer when the usual best practice guidelines are followed. However, the aforementioned studies do not provide a detailed analysis of the results in terms of best practice guidelines to be used in RANS simulations, which are, at this time, the only affordable technique at industrial scale.

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Nomenclature

Abbreviations

CFL	Courant–Friedrichs–Lewy
DNS	Direct Numerical Simulation
GEN-IV	Generation IV
LBE	lead–bismuth eutectic
LES	Large-Eddy Simulation
LMR	liquid metal reactor
RANS	Reynolds-averaged Navier–Stokes
SGS	subgrid-scale
WALE	wall-adapting local Eddy-viscosity

Greek Symbols

α	molecular heat diffusivity [m^2/s]
α_t	turbulent heat diffusivity [m^2/s]
$\Delta x, \Delta y, \Delta z$	grid spacings in the x, y, z directions [m]
Δ	mean grid spacing $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ [m]
δ	half-width of the channel [m]
ϵ	turbulent dissipation [m^2/s^3]
η	Kolmogorov scale $\eta = (\nu^3/\epsilon)^{1/4}$ [m]
η_T	Corrsin scale $\eta_T = \eta Pr^{-3/4}$ [m]
γ	mesh stretching parameter
κ	Von Karman constant
μ	dynamic viscosity [Pa.s]
ν	molecular kinematic viscosity [m^2/s]
ν_t	turbulent kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
τ	total shear stress $\tau = \tau_M + \tau_t + \tau_{SGS}$ [Pa]
τ_M	molecular shear stress [Pa]
τ_t	turbulent shear stress [Pa]
τ_w	wall shear stress [Pa]
τ_{SGS}	SGS shear stress [Pa]
θ	temperature difference $\theta = T - T_w$ [K]
ζ	wall-normal coordinate in computational space

Operators, subscripts and superscripts

$(\cdot)'$	fluctuations $(\cdot)' = (\cdot) - \overline{(\cdot)}$
$(\cdot)^+$	dimensionless quantities normalised using $\bar{u}_\tau, \nu, \overline{T}_\tau$
$\overline{(\cdot)}$	Reynolds-averaged quantities
$\overline{(\cdot)}_{\text{rms}}$	root mean square fluctuations $\overline{(\cdot)}_{\text{rms}} = \left(\overline{(\cdot)'^2}\right)^{\frac{1}{2}}$

Roman symbols

$\bar{k} - \bar{\epsilon}$	RANS model based on transport equations for the turbulent kinetic energy and the dissipation
$\bar{k} - \bar{\omega}$	RANS model based on transport equations for the turbulent kinetic energy and the inverse time-scale ω
$\overline{T}_b$	bulk temperature $\overline{T}_b = \frac{1}{\bar{u}_b \delta} \int_0^\delta (\overline{T}\bar{u}) dy$ [K]
$\overline{T}_\tau$	friction temperature [K]
$\bar{u}_\tau$	friction velocity $\bar{u}_\tau = \sqrt{\frac{\tau_w}{\rho}}$ [m/s]
$\bar{u}_b$	averaged bulk velocity $\bar{u}_b = \frac{1}{\delta} \int_0^\delta \bar{u} dy$ [m/s]
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$	unit vectors in the x, y and z directions
$\mathbf{F}$	forcing term of the momentum equation [m/s^2]
C	SGS model coefficient
c	specific heat [J/kg/K]
dP_f/dx	pressure gradient forcing [m/s^2]
k	heat conductivity [W/m/K]
L_x, L_y, L_z	domain size in the x, y, z directions [m]
N_x, N_y, N_z	number of grid points in the x, y, z directions
Nu	Nussel number
P	reduced pressure $P = p/\rho$ [m^2/s^2]
p	pressure [Pa]
Pe	Peclet number $Pe = PrRe$
Pe_t	turbulent Peclet number $Pe_t = \frac{\nu_t}{\nu} Pr$
Pr	molecular Prandtl number $Pr = \frac{\nu}{\alpha}$
Pr_t	turbulent Prandtl number $Pr_t = \frac{\nu_t}{\alpha_t}$
$Pr_{t\infty}$	turbulent Prandtl number far from the wall in the Weigand correlation
q	total heat flux $q = q_M + q_t$ [W/m^2]
q_M	molecular heat flux [W/m^2]
q_t	turbulent heat flux [W/m^2]
q_w	wall heat flux [W/m^2]
Re	bulk Reynolds number $Re = \frac{\bar{u}_b (2\delta)}{\nu}$
Re_τ	friction Reynolds number $Re_\tau = \frac{\bar{u}_\tau \delta}{\nu}$
S_θ	source term in the energy equation for θ [W/m^3]
T	temperature [K]
T_w	wall temperature [K]
u, v, w or u_1, u_2, u_3	Cartesian velocity components [m/s]
x, y, z	Cartesian coordinates in the streamtwise, wall-normal and spanwise directions [m]
y_1	wall-normal coordinate of the first grid point [m]

For RANS modeling, the main issue is the determination of the effective turbulent heat diffusivity in the core flow. This is usually achieved by using a turbulent Prandtl number close to unity. However, DNS simulations [3,4] showed that the turbulent Prandtl number is higher than unity at low Prandtl. Bricteux et al. [6,7] obtained a value of $Pr_t \approx 2$ at moderate Reynolds numbers and $Pr = 0.01$. Several correlations to obtain flow dependent turbulent Prandtl numbers have been proposed in the literature, e.g. [8–11]. Baumann et al. [12] assessed a few Pr_t correlations on various cases including moderate Reynolds number channel flows and annular pipe flows. In the channel flows, the results are improved when using a good correlation but, in more complex flows, the best results were obtained using a constant $Pr_t = 0.9$ because of the inaccuracies of the eddy-viscosity computed by the turbulence model. The sensitivity of heat transfer in liquid metal to the value of Pr_t was studied by several authors in various configurations but using a uniform Pr_t: Cheng and Tak [13] investigated the case of a subchannel and Thiele and Anglart [14] performed a sensitivity analysis of the LBE flow in an annulus around a single heated rod. A constant value of $Pr_t = 0.9$ was also used by Chandra and Roelofs [15] for subchannel simulations with liquid metal. One of the objectives of this paper is to further assess existing Pr_t correlations and to study the influence of Pr_t in RANS simulations. It should be mentioned that more complex approaches involving additional transport equations are also proposed to simulate heat transfer in liquid metal (see e.g. Shams et al. [16] and Manservisi and Menghini [17]). Such models may be required in flows where buoyancy plays an important role, see Grötzbach [2].

When wall-resolved RANS simulations cannot be used, as is sometimes the case in industrial applications because one looks for a "quick insight" into the problem or because the grid is unaffordable, a second issue concerns the location of the first grid point for such un-resolved simulations and the wall-functions (velocity and temperature) to be used. For liquid metals with $Pr \leqslant 0.01$, which encompasses the LBE for certain temperatures and the sodium in general, it has been shown (e.g., Bricteux et al. [6]) that the laminar sublayer for temperature extends up to $y^+ \approx 60$. This makes the possible location of the first grid point to be in a very narrow band of y^+, providing the lower bound of the log-profile for velocity could be considered at such low y^+ value ($y^+ \approx 60$). For higher Prandtl numbers or for a less stringent first grid point placement, a law for the temperature is required. A common