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# Laminar and steady free convection in power-law fluids from a heated spheroidal particle: A numerical study

A.K. Gupta, C. Sasmal [1], M. Sairamu [2], R.P. Chhabra *

*Department of Chemical Engineering, Indian Institute of Technology, Kanpur 208016, India*

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ABSTRACT

In this work, the effects of aspect ratio and shear-dependent viscosity on the laminar free convection heat transfer from a heated spheroid immersed in unbounded quiescent power-law fluids have been investigated. In particular, the coupled momentum and energy equations have been solved numerically over the following ranges of the pertinent governing parameters: Grashof number, $10 \leqslant Gr \leqslant 10^5$; Prandtl number, $0.72 \leqslant Pr \leqslant 100$; power-law index, $0.3 \leqslant n \leqslant 1.5$ and aspect ratio, $0.2 \leqslant e \leqslant 5$. Detailed structures of the flow and temperature fields in the vicinity of the spheroid are visualized in terms of the streamline and isotherm patterns, whereas the gross flow and heat transfer phenomena are resolved in terms of the local Nusselt number and its surface averaged value and drag coefficient ($C_D$). Broadly speaking, shear-thinning fluid behaviour ($n < 1$) facilitates heat transfer whereas shear-thickening ($n > 1$) impedes it in comparison to that seen in Newtonian fluids ($n = 1$) under otherwise identical conditions. At fixed values of the Grashof number ($Gr$), Prandtl number ($Pr$) and power-law index ($n$), the value of Nusselt number gradually increases as the spheroid shape progressively passes from the oblate ($e > 1$) to the prolate ($e < 1$) configurations via the spherical shape ($e = 1$). The reverse trend occurs, however, for the drag coefficient ($C_D$). Finally, the present values of the average Nusselt number and drag coefficient are correlated using a simple analytical form based on a general composite parameter proposed for power-law fluids. The paper is concluded by presenting some comparisons with the limited previous analytical and experimental results available in the literature which are limited to Newtonian fluids.



## 1. Introduction

Current interest in studying free convection heat transfer from various objects of two-dimensional (such as long cylinders of circular and non-circular cross-section) and three-dimensional (spheres, hemispheres and spheroids) shapes immersed in quiescent fluids stems both from fundamental and pragmatic considerations. From a theoretical standpoint, since in this regime, the motion is caused solely by the temperature-dependent density, not only the momentum and energy equations are coupled, the shape and orientation of the object also exert a significant influence on the resulting flow and temperature patterns in the close proximity of the object which eventually impact on the rate of heat transfer. Similarly, reliable values of the convective heat transfer coefficient are frequently needed in the sizing of process equipment entailing the heating/cooling of slurries, melting of polymeric melts, food processing applications, etc. Since in most real-life engineering applications, heat transfer occurs in the so-called mixed convection regime, the overall heat transfer draws varying contributions from the free- and forced-convection mechanisms. The relative importance of the two contributions is governed by the value of the familiar Richardson number, $Ri$, defined as the ratio of the Grashof number ($Gr$) and Reynolds number ($Re$) as $Ri = Gr/Re^2$. Thus, the two limiting cases of $Ri \to 0$ and $Ri \to \infty$ correspond to the pure forced- and free-convection regimes respectively. Naturally, as the value of the Reynolds number is reduced, as is the case for viscous Newtonian and non-Newtonian fluids, the importance of free convection progressively increases.

In view of the preceding discussion, over the years, significant research effort has been expended in studying free convection heat transfer from variously shaped single and multiple objects in Newtonian fluids like air and water. The bulk of the literature in this field has been summarized, amongst others by Jaluria et al. [1,2], Martynenko and Khramstov [3], Fand et al. [4,5], Churchill and co-workers [6,7], Morgan [8] and more recently by Eslami and co-workers [9]. An examination of the available literature reveals

* Corresponding author. Tel.: +91 512 2597393; fax: +91 512 2590104.
*E-mail address:* chhabra@iitk.ac.in (R.P. Chhabra).
[1] Present address: Department of Chemical Engineering, Monash University, Clayton, Australia.
[2] Present address: R & D Office, Hindustan Petroleum Corporation Ltd., Bangalore.

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**Nomenclature**

| | |
|---|---|
| $A$ | surface area, $m^2$ |
| $a$ | semi-axis normal to the direction of gravity, m |
| $b$ | semi-axis along the direction of gravity, m |
| $e$ | aspect ratio, $(= a/b)$, dimensionless |
| $C_D$ | total drag coefficient, $C_D = \frac{2F_D}{\rho_\infty U_c^2 \pi b^2}$, dimensionless |
| $C_{DP}$ | pressure component of drag coefficient, $C_{DP} = \frac{2F_{DP}}{\rho_\infty U_c^2 \pi b^2}$, dimensionless |
| $C$ | thermal heat capacity of fluid, J $kg^{-1}$ $K^{-1}$ |
| $D_\infty$ | diameter of the outer domain, m |
| $F_D$ | total drag force, N |
| $F_{DP}$ | pressure component of drag force, N |
| $g$ | acceleration due to gravity, m $s^{-2}$ |
| $Gr$ | Grashof number based on the length scale of $2b$, dimensionless |
| $Gr_b$ | Grashof number based on the length scale of $b$, dimensionless |
| $h$ | heat transfer coefficient, W $m^{-2}$ $K^{-1}$ |
| $I_2$ | second invariant of the rate of strain tensor, $s^{-2}$ |
| $k$ | thermal conductivity of fluid, W $m^{-1}$ $K^{-1}$ |
| $m$ | power-law consistency index, Pa.$s^n$ |
| $n$ | power-law index, dimensionless |
| $n_s$ | unit normal vector |
| $Nu$ | average Nusselt number based on the characteristic length of $2b$, dimensionless |
| $Nu_{2a}$ | average Nusselt number based on the characteristic length of $2a$, dimensionless |
| $Nu_{\sqrt{A}}$ | average Nusselt number based on $\sqrt{A}$ as the length scale, dimensionless |
| $Nu_\theta$ | local Nusselt number, dimensionless |
| $Nu_\infty$ | conduction limit of the Nusselt number, dimensionless |
| $P$ | pressure, dimensionless |
| $Pr$ | Prandtl number based on the characteristic length of $2b$, dimensionless |
| $Pr_b$ | Prandtl number based on the length $b$, dimensionless |
| $Ra$ | Rayleigh number based on the length scale of $2b$, dimensionless |
| $Ra_{2a}$ | Rayleigh number based on the length scale of $2a$, dimensionless |
| $Ra_{\sqrt{A}}$ | Rayleigh number based on $\sqrt{A}$, dimensionless |
| $T$ | temperature, K |
| $\Delta T$ | temperature difference $(=T_W - T_\infty)$, K |
| $\mathbf{U}$ | velocity vector, dimensionless |
| $U_c$ | characteristic or reference velocity, m $s^{-1}$ |
| $U_X$, $U_Y$ | $x$- and $y$-components of the velocity, dimensionless |
| $X$, $Y$ | Cartesian coordinates, dimensionless |

*List of Greek symbols*

| | |
|---|---|
| $\beta$ | coefficient of volume expansion, $K^{-1}$ |
| $\delta$ | distance between two grid points on spheroid surface, m |
| $\varepsilon$ | components of the rate of strain tensor, $s^{-1}$ |
| $\rho$ | density of fluid at temperature $T$, kg $m^{-3}$ |
| $\xi$ | non-dimensional temperature, dimensionless |
| $\tau$ | extra stress tensor, Pa |
| $\theta$ | location on the surface of the spheroid, degree |
| $\psi$ | surface contour, m |
| $\nabla$ | Del operator, dimensionless |

*Subscripts*

| | |
|---|---|
| $w$ | spheroid surface condition |
| $\infty$ | corresponds to far away condition |

that the case of a sphere and a circular cylinder in various configurations have occupied the centre stage, followed by elliptical cylinders and other two-dimensional shapes like bars of square and non-square cross-sections [10,11] in Newtonian fluids. In contrast, many food-stuffs, catalyst particles, polymeric and pharmaceutical and agricultural products are of granular form but not necessarily spherical in shape. Undoubtedly, the study of free convection from a sphere has yielded valuable insights not only about the underlying processes but also about the different flow regimes and transitions from one regime to another. Consequently, a wealth of information is now available on this subject as far as a sphere in Newtonian media is concerned. On the other hand, the use of spheroidal-shaped objects affords the possibility of delineating the influence of shape and orientation on free convection heat transfer by simply varying the aspect ratio of a spheroid. Indeed, there has been a spurt in studying momentum and heat transfer characteristics of heated spheroidal particles in confined [12,13], unconfined [14], Newtonian [15–21] and power-law fluids [22–24]. However, most of these are restricted to the forced convection regime in the steady flow region except for the unsteady case considered by Juncu [15].

In contrast, very little is known about the free convection heat transfer from spheroidal particles, even in Newtonian fluids, let alone in power-law fluids. Most of the developments in this area are based on the approximate boundary layer analysis aided by dimensional considerations. Thus, for instance, Raithby et al. [25,26] reported experimental results on the average Nusselt number from spheroids in air and they found it necessary to incorporate the curvature effects (often ignored in boundary layer treatments) and/or to account for turbulence to obtain satisfactory match between their data and predictions. Similarly, based on experimental results, Yovanovich and co-workers [27–29] have attempted to identify a characteristic linear dimension for spheroids, cubes, cones, two spheres joined together in an endeavour to consolidate the average Nusselt number results for scores of shapes. For instance, Yovanovich [28] has argued that the use of $\sqrt{A}$ (where $A$ is the surface area available for heat transfer) as the characteristic linear dimension in the definitions of the Nusselt and Grashof numbers does lead to unification of the experimental data for a range of shapes. They have attempted to fit the following generic form of expression:

$$Nu_{\sqrt{A}} = Nu^{\infty}_{\sqrt{A}} + F(Pr)G_{\sqrt{A}}Ra^{1/4}_{\sqrt{A}} \tag{1}$$

In Eq. (1), $Nu^{\infty}_{\sqrt{A}}$ is the conduction limit ($Ra_{\sqrt{A}} \to 0$) and indeed it turns out to be only a weak function of body shape. For example, the value of $Nu^{\infty}_{\sqrt{A}}$ ranges from 3.545 for a sphere to 3.39 for cubes, 3.44 for horizontal and vertical cylinders, 3.34–3.57 for prolates and oblates. Undoubtedly, this level of consolidation is quite acceptable in process engineering calculations. Now retuning to the second term on right hand side of Eq. (1), in most instances, experiments are often performed in air (i.e., fixed value of $Pr$ = 0.70 or so), the function $F(Pr)$ is introduced here to generalize the results to the other values of Prandtl number. The following form due to Churchill and Churchill [6] has gained wide acceptance in the literature hence Yovanovich and co-workers also [27–29] recommended it:

$$F(Pr) = \frac{0.670}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{4/9}} \tag{2}$$