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Higher order compact computations of transient natural convection in a deep cavity with porous medium

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ABSTRACT

In this paper, an extension of the fourth order compact scheme on nonuniform grids (Pandit et al. (2007) [28]) is proposed for solving two dimensional (2D) unsteady natural convection flows in a rectangular cavity (with different aspect ratios) filled with a fluid saturated porous medium. The bottom wall of the cavity is uniformly and non-uniformly heated and the top wall is adiabatic while the vertical walls are cold maintained at constant temperature. We have used streamfunction (ψ)–vorticity (ζ) formulation of Navier–Stokes equations with the consideration of Brinkmann-extended Darcy model to simulate the momentum transfer in the porous medium. The streamfunction–vorticity and the energy equations are all solved as a coupled system of equations for the five field variables consisting of streamfunction, vorticity, two velocities and temperature. In this $\psi-\zeta$ formulation, the temperature gradient source term also has been treated as fourth order compact. The higher order compact scheme adopted in the present study yields consistent performance for a wide range of key parameters e.g. Rayleigh number *Ra* (from 10^3 to 10^8), Darcy number *Da* (from 10^{-5} to 10^{-3}). Results are presented in the form of streamline and isotherm plots as well as the plots of Nusselt number at the heat source surface under different conditions. The present scheme is not only robust as evidenced from computations at higher *Ra*, but also accurate as is seen from comparisons with reliable existing results.

1. Introduction

Natural convection in a square cavity has been studied widely due to a large number of engineering and technical applications and has attracted many researchers' interest. In recent years, there has been considerable attention to study the flow in a thermally driven square cavity through porous media due to large number of technical applications such as packed sphere beds, chemical catalytic reactors, grain storage, geothermal reservoirs, solidification of casting, crude oil production etc. In this context, the readers are refereed to [2–5,9–12,17,18,21,23] (as well as the references therein). Experimentally, fluid flow deviations from Darcy's law have long been observed. Various terms, such as Darcy flow, non-Darcy flow, turbulent flow, inertial flow, high velocity flow, etc., have been used to describe this behavior. For very high velocities in porous media, inertial effects can also become significant and in such situations, non-darcy behavior is important for describing fluid flow. Vafai and Tien [20] presented an in-depth analysis of boundary and inertia effects. It may be mentioned here that Amiri and Vafai [8] conducted a comprehensive transient analysis of incompressible flow through a packed bed, taking into the account the inertial and viscous forces. They found that excluding the inertial effects would shorten the time needed to reach steady-state. This was expected since the absence of the inertia effects would cause a higher flow rate, which results in a higher rate of heat transfer between the fluid and the solid, achieving steady state conditions in a relatively shorter time. In addition, for a given flow driving force (pressure gradient or buoyancy) the results have shown that each of the three effects, i.e. inertia, boundary and velocity-square, reduces the heat transfer rate [22]. The buoyancy driven convection in a porous cavity heated differentially in the horizontal side has been analyzed by Walker and Homsy [10] by using different techniques and found fairly good agreement with each other as well as experimental results. However, most of the existing studies in the literature dealt primarily with the mathematical simplifications based on Darcy's law, which neglect the effects of the solid boundary, inertial forces, and variable porosity on flow through porous media. When relaxing some assumptions one gets more complex models. For instance, assuming that the frictional effects in the fluid are not negligible one gets the

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Nomenclature

AR	aspect ratio (H/L)
Da	Darcy number
Ra	Rayleigh number
Pr	Prandtl number
g	acceleration due to gravity (ms^2)
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
K	permeability of the porous medium
J	Jacobian
H	cavity height (m)
L	cavity width (m)
p	pressure (Pa)
P	dimensionless pressure
T^*	temperature (K)
T	dimensionless temperature
T_0	reference temperature (K)
T_h	temperature of hot bottom wall (K)
T_c	temperature of cold vertical wall (K)
u, v	dimensionless velocities in x, y directions respectively
x, y	dimensionless distances along x and y coordinate respectively
t	time
h, k	step lengths in horizontal and vertical directions respectively

Greek symbols

ν	kinematic viscosity $(\mathrm{m}^2/\mathrm{s})$
ρ	density $(\mathrm{kg/m}^3)$
α^*	thermal diffusivity $(\mathrm{m}^2/\mathrm{s})$
β	thermal expansion coefficient (1/K)
ψ	stream function
ζ	vorticity
ξ	horizontal coordinate in a unit square in computational plane
η	vertical coordinate in a unit square in computational plane

Subscripts

i, j	cell faces

Superscript

n	time level

Brinkman model. Further, if one relaxes the hypothesis of negligible inertial effects one gets a series of nonlinear models, depending on the way interaction forces are modelled. The aforesaid discussions motivate to study natural convection without considering the Forchheimer inertia term.

Many numerical methods, including finite difference, finite element, finite volume, and lattice boltzman methods have been used to investigate the steady natural convection in a square cavity. Most of these numerical schemes are either first-order or second-order accurate in space, particularly the central difference ones have been used in a large number because of their straightforwardness in application. In keeping with this trend, in most of the previous attempts to tackle the problem of natural convection, the schemes were at most second-order accurate in space. Also, whenever there has been attempts to solve for the transient flows, they are confined invariably to uniform space grids. However, a very little work has been done for the case of unsteady and transient flow situations.

Lauriat and Prasad [23], and Pop et al. [9,29] have made fruitful contributions to this area. However, their scheme could not fully exploit the advantages associated with using nonuniform grids, particularly that of mesh grading to resolve smaller scales in the regions of large gradients in the physical domain. The model commonly used of a porous cavity with both the vertical walls maintained at constant temperatures, while the horizontal walls are adiabatic. Some investigations have been made in the past [14,15] to focus on natural convection in porous medium due to uniform heating from below. Recently, Basak et al. [16] have studied the uniform and nonuniform heating of the bottom wall. They have considered the Brinkmann-extended Darcy model i.e. by neglecting the inertia term from Forchheimer Darcy model to study natural convection using Penalty-Galerkin finite-element method. Almost all of these works presented results for $Ra = 10^3$ to $Ra = 10^6$. However, note that most methods are available to solve this problem in moderate Rayleigh numbers until 10^6, and therefore it is still necessary to develop accurate and efficient numerical methods suitable for a wide range of Rayleigh numbers. To the best of our knowledge, no studies have been found in the literature on transient natural convection in a fluid saturated porous medium using higher order compact scheme on nonuniform grids. This motivates to use higher order compact scheme for heat transfer problems, where the main objective is to obtain the transient solutions for different aspect ratios and also to examine the numerical performance of our proposed scheme. In computational fluid dynamics field of study, high-order compact formulations are becoming more popular as compact formulations provide more accurate, efficient and stable solutions in a compact stencil [24,27,28,31].

In this work, we have studied the problem on natural convection flow in a rectangular cavity (with aspect ratio 0.5, 1, 2, 4 and 6) filled with fluid saturated porous medium by considering the Brinkmann-extended Darcy model with our proposed scheme which is an extension of the fourth order accurate compact scheme [28] on nonuniform grids. In this extension, the temperature gradient source term in the vorticity equation is not explicitly known and has been discretized as fourth order compact and smoothly integrated to the solution procedure. Another attractive feature of the computation is the use of hybrid BiCGStab [6,30] algorithm for solving the resultant linear algebraic equations and this improves the convergence behavior of the algorithm. As expected, the results obtained using higher order compact scheme are accurate and even excellent agreement of our coarse grid results with the results available in the literature.

The remainder of this paper is organized as follows. Section 2 gives formulation of the problem. Section 3 describes the discretization of the governing equations with related issues. Section 4 analyzes the solution procedure of algebraic systems. Section 5 deals with the results and discussions and Section 6, the conclusions.

2. Problem formulation

2.1. The problem

The problem considered in this paper is the two-dimensional transient natural convection of fluid in a rectangular enclosure (filled with a fluid saturated porous medium) of height H and width L having aspect ratios $(AR = \frac{H}{L})$ 0.5, 1.0, 2.0, 4.0 and 6.0. The flow is driven by a uniform and nonuniform temperature profile applied along the bottom wall of the enclosure, as illustrated in