International Journal of Heat and Mass Transfer 75 (2014) 656–667

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Phase-lag heat conduction in multilayered cellular media with imperfect bonds

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ARTICLE INFO

Article history:
Received 16 December 2013
Received in revised form 28 March 2014
Accepted 31 March 2014
Available online 6 May 2014

Keywords:
Functionally graded material
Cellular solid
Porous multilayered composites
Heat conduction
Phase-lag theory
Thermal wave

ABSTRACT

We present a theoretical framework to study the thermal responses of one-dimensional multilayered systems, functionally graded solid media, and porous materials. The method for thermal analysis resorts to non-Fourier heat conduction theories including three-phase-lag, dual-phase-lag, and hyperbolic heat conduction. The graded media are modeled as multilayered systems displaying finite numbers of layers. For each homogenous layer, the differential equations of heat conduction describing the wave-like three-phase-lag are solved in closed-form in the Laplace domain. Solutions accounting for proper interfacial and boundary conditions are first presented to describe the thermal behavior of heterogeneous solids and porous media. Transient temperature and heat flux are obtained in time domain via fast Laplace inversion. We then apply the solutions obtained with each heat conduction theory to one-dimensional media and compare their thermal behavior. Finally, maps are presented to visualize the thermal responses of cellular materials, functionally graded cellular materials, and multilayered systems. For the latter, particular attention is devoted to investigate the impact of key attributes defining graded media, such as layer bond imperfections and material heterogeneity.

1. Introduction

The governing equations of heat conduction are generally described by the heat flux-temperature gradient and the first-law of thermodynamics. If we use the conventional Fourier equation to solve the heat conduction problem, two problems emerge. First, the resulting differential equation, which is parabolic, turns out to predict an infinite thermal wave speed that is physically unrealistic. Second, for very low temperature, short-pulse thermal heating, as well as for micro-temporal and spatial scale applications, the results are not aligned with experimental observations [1]. To reconcile these discrepancies, several non-Fourier heat conduction theories have been introduced. Cattaneo and Vernotte (C–V) first introduced a thermal relaxation time in the model with a hyperbolic heat conduction and finite thermal wave speed [2]. Although valuable, the C–V model leads to results that cannot accurately describe experimental data, mainly because the microstructural effects in the heat transport process are overlooked. As a result, Tzou [3,4] proposed a dual-phase-lag (DPL) model that could also account for microstructural interactions such as phonon-electron and phonon scattering via the phase-lag of temperature gradient, and fast transient effect of thermal waves through phase-lag of heat flux. A three-phase-lag (TPL) heat conduction theory was later proposed by Choudhuri [5] to encompass all previous theories for non-Fourier heat conduction. The fractional derivative of the phase-lag heat conduction was also introduced by Ezzat et al. [6,7]. The use of phase-lags of heat flux, temperature gradient, and thermal displacement gradient in the TPL model is important to understand several phenomena, such as bioheat transfer in living tissues, exothermic catalytic reactions, and harmonic plane wave propagation.

The study of heat conduction with non-Fourier approaches has been the subject of several theoretical investigations [8–13]. These approaches attempt to predict phenomena that cannot be captured by classical Fourier theories of heat conduction. For example, Ramadan [14] presented a semi-analytic solution for heat conduction in a multilayered composite by using DPL theory. The TPL phase-field system for thermal flux was studied by Miranville and Quintanilla [15]. Wang et al. [16,17] studied the non-Fourier heat conduction in carbon nanotubes based on the concept of thermomass. The transient temperature field of the DPL model around a partially insulated crack was studied by Hu and Chen [18]. Recently, Afrin et al. [19] employed the DPL model for heat conduction analysis

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.03.090
0017-9310/

in a gas-saturated porous medium subjected to a short-pulsed laser heating.

Other works in literature aim at studying the accuracy of thermal induced responses obtained with non-Fourier heat transport models. Babaei and Chen [20] investigated the generalized coupled thermopiezoelectric response of a functionally graded (FG) cylinder using the finite element method. The coupled and uncoupled transient thermopiezoelectric behavior of a one-dimensional (1D) FG rod was investigated by Akbarzadeh et al. [21,22]. Hosseini zad et al. [23] used the classical and generalized coupled thermoelasticity to describe the behavior of thermoelastic waves at the interfaces of a layered medium. Banik and Kanoria [24] dealt with the TPL thermoelastic interactions in an FG unbounded medium subjected to periodically varying heat sources. Akbarzadeh et al. [25,26] employed the higher-order shear deformation theory to study the classical coupled and uncoupled thermoelasticity of FG thick plates.

In automotive, naval, and aerospace applications, sudden temperature changes are commonly experienced by structural materials, such as multi-phase and fiber-reinforced composites [27]. An accurate thermal analysis, as studied in [28–35], is thus essential to predict the level of thermal-induced deformation in composite materials. Yang and Shi [36], for example, established a stability test for heat conduction in a 1D multilayered solid. For a multilayered hollow cylinder, Jain et al. [37] presented a closed-form expression containing a double-series for time-dependent asymmetric heat conduction. An exact solution for transient heat conduction in cylindrical multilayered composites was presented by Amiri Delouei et al. [38]. Akbarzadeh and Chen [39,40] theoretically studied the effect of steady-state hygrothermal loading on the magnetoelectroelastic responses of homogeneous and heterogeneous media.

Laminated composites often contain imperfections, such as small voids and defects, at the interfaces where cracks initiate and propagate. For this reason, the multiphysics of imperfectly bonded composites has become a subject of study [41–45]. Duan and Krihaloo [46], for example, studied the effect of imperfect bonding between the inclusions and matrix on the effective thermal conductivity of heterogeneous media. Hatami-Marbini and Shodja [47] studied the stress field of multi-phase inhomogeneity systems with perfect/imperfect interfaces under uniform thermal and far-field mechanical loading.

The mismatch of thermal properties between bonded layers in laminated composites causes structural failure. This is one reason for which functionally graded materials (FGMs) with continuous transition of material constituents have been introduced. FGMs enable to greatly reduce thermal stresses and stress concentrations [48–50]. Since their appearance, FGMs have been the subject of intense research [51–59]. An approximate solution was developed by Ishigura et al. [60] and Tanigawa et al. [61] for transient Fourier heat conduction in an FGM plate by using piecewise homogeneous layers. A closed-form asymptotic solution was later obtained by Jin [62] for the short-time temperature field via a multilayered material model. Zhou et al. [63] derived exact solutions for the transient heat conduction in an FG strip in contact with a fluid. Using the conjugate gradient method, the inverse hyperbolic heat conduction problem in FG cylinders was solved by Yang et al. [64] to estimate the heat flux from the temperature measurements. A theoretical framework was also proposed by Wang [65] to analyze the transient thermal analysis in FG hollow cylinders via the state space approach as well as the initial parameter method.

All of the forementioned works dealt with solid media; nonetheless, porous materials, such as foams and lattices, can be effectively used for thermal management. Due to the increasingly growing number of applications, such as sandwich panels, heat exchangers, and heat shields [66,67], several theoretical and experimental studies have been proposed to understand their thermal behavior [68–71]. Leong and Li [72] obtained the effective thermal conductivity of porous structures via a unit cell model. Sadeghi et al. [73] designed a test bed to measure thermal conductivity and thermal contact resistance of metallic foams under compressive loads. A three-dimensional finite element model for characterizing the elastic, dielectric, and piezoelectric properties of piezoelectric foam structures was recently developed by Challagulla and Venkatesh [74]. Furthermore, recent advances in manufacturing have enabled the engineering of FG foams, an element that further motivates research on their thermal behavior [75,76]. For instance, Zhu et al. [77] considered the problem of minimizing the maximum temperature of a structure insulated by FG metallic foams under transient heat conduction.

Thermal analysis of heterogeneous solid and porous structures requires an accurate prediction of steady-state and transient temperature fields. This work theoretically investigates the non-Fourier heat conduction in graded solid media with application for the first time to cellular materials. It extends the DPL heat conduction analysis in a heterogeneous medium conducted by Akbarzadeh and Chen [78]. The paper provides a semi-analytic solution via Bessel functions and Laplace transform for 1D graded solid and porous media with perfect/imperfect bonding interfaces for TPL, DPL, and C–V heat conduction theories. The semi-analytic solutions obtained in this work enable modeling FG structures with an arbitrary material profile and predict their thermal properties. Closed-form solutions are obtained in terms of Bessel functions in Laplace domain in Section 3. Section 4 presents the thermal responses of 1D media obtained with alternative heat conduction theories. Finally, the effect of FGM profile, bonding imperfection, and heterogeneity is visualized into maps presented in Sections 5 and 6.

2. Problem definition and governing equations

This section reports the governing equations of non-Fourier heat conduction in a one-dimensional (1D) graded medium. Fig. 1 shows its material gradients in a general 1D coordinate system $\vec{x}$, where the number of layers is N. The position of the inner and outer surfaces is x_i and x_o; x_n $(n = 1, 2, \ldots, N)$ is the inner surface of the nth layer with $x_1 = x_i$ and $x_{N+1} = x_o$. The heterogeneous medium is initially at ambient temperature T_0, and is here approximated by a piecewise profile consisting of N homogenous layers.

We can describe the general phase-lag heat conduction for each layer as [5,79]:

$$\vec{q}^{(n)}(\vec{x}, t + \tau_q^{(n)}) = -\left[K^{(n)}\vec{\nabla}T(\vec{x}, t + \tau_T^{(n)}) + K^{*(n)}\vec{\nabla}\upsilon(\vec{x}, t + \tau_\upsilon^{(n)})\right] \quad (1)$$

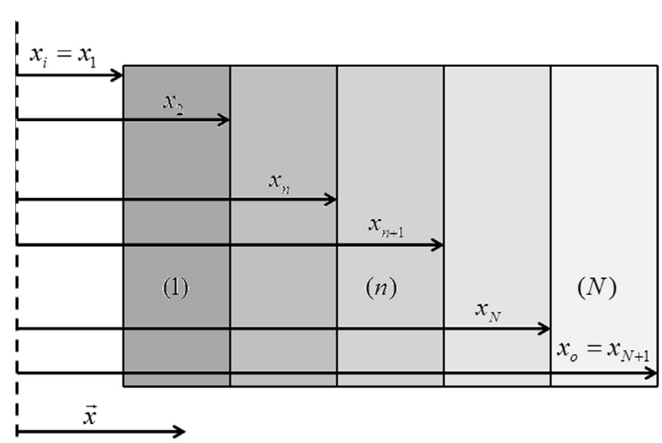

Fig. 1. Multilayered medium in a general 1D coordinate system.