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Exact solutions for two-dimensional laminar flow over a continuously stretching or shrinking sheet in an electrically conducting quiescent couple stress fluid

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ABSTRACT

A mathematical study of heat and flow of couple stress MHD fluids over permeable stretching/shrinking surfaces is undertaken in the present paper. Exact solutions for both flow and temperature fields under a boundary layer approach are targeted. In the absence of couple stress field the results completely collapse onto the special cases available in the literature. Obtained closed form solutions provide valuable knowledge for the velocity and temperature profiles as well as skin friction coefficient and Nusselt number. Dissimilar to the already known for a noncouple Newtonian fluid, double solutions exist over a stretching sheet, and triple solutions are present over a shrinking sheet, if couple stress effects are taken into consideration.

1. Introduction

Pursuing the pioneering studies of Sakiadis [3] and Crane [4], the flow over continuously stretching or shrinking surfaces and its numerous physical features have been investigated by many researchers, amongst them are [5–10,1,11,2]. The significance of this problem arises from many real life applications, such as polymer extrusion, drawing of plastic films and wires, glass fiber and paper production, manufacture of foods, crystal growing, cooling of metallic plate in a bath, liquid films in condensation process, etc., see Fisher [12]. The present work is also devoted to stretching or shrinking sheet problems, but when the boundary layer flow evolves in accordance with the model of couple stress fluid.

Couple stress fluid theory is just generalization of the classical Newtonian theory of fluids permitting polar effects such as the presence of couple stresses and body couples, see [13,14]. One similar theory was also suggested in [15]. Apart from the practical applications in the field of biomechanics and in the fluid models of the mixture of Newtonian and non-Newtonian immiscible fluids, petroleum and chemical industries, geohydrology, extraction of geothermal energy and medicine are a few well-studied problems concerning couple stress fluids having technological importance. When blood is represented by a couple stress fluid model, the effects of an axially symmetric mild stenosis on the flow of blood were examined in [16,17]. In a series of work, [18,19] investigated rotor bearing system when the bearings are lubricated with couple stress fluid. A similar work was implemented in [20] when the journal bearings are further influenced by surface roughness. [21] considered the effect of induced magnetic field over peristaltic flow of a couple stress fluid occurring in an asymmetric channel. An approximate closed from solution was recently presented in [22] for the partial journal bearings with couple stress fluids.

Motivated by the aforementioned importance of couple stress fluids, the flow and heat transfer analysis are the essential ingredients of the current study, when the motion of the fluid takes place along stretching or shrinking surfaces. The aim is to obtain exact closed form solutions of the physical model accounting for the permeability as well as MHD effects. Such type of analysis for regular fluids was already given in [23–26]. Quite distinct from the nonpolar regular Newtonian fluids, it has been found that the couple stress fluids lead to existence of double solutions over stretching and triple solutions over shrinking sheets.

The following is pursued in the rest of the paper. Formulation of the couple stress fluid flow is given in Section 2. Exact solutions are then presented in Section 3 both for the flow and temperature fields. Section 4 contains results and discussions. The concluding remarks eventually are drawn in Section 5.

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2. Formulation of the problem

Consider a steady, two-dimensional laminar flow over a continuously stretching or shrinking sheet in an electrically conducting quiescent couple stress fluid. A uniform external magnetic field of strength B_0 is supposed to act in the direction perpendicular to the sheet. The coordinates x and y are used such that x is along the surface of the sheet, while y is taken as normal to it. The governing equations are then given by

$$\begin{aligned} &u_x + v_y = 0, \\ &uu_x + vu_y = \nu u_{yy} - \frac{\eta_0}{\rho} u_{yyyy} - \frac{\sigma B_0^2}{\rho} u, \\ &uT_x + vT_y = \frac{k}{\rho c_p} T_{yy}, \end{aligned} \tag{2.1}$$

supplemented with the subsequent boundary conditions

$$\begin{aligned} &u(x,0) = dcx, \quad v(x,0) = v_w, \quad u(x,\infty) = 0, \quad u_{yy}(x,\infty) = 0, \\ &v_{yy}(x,\infty) = 0, \\ &T(x,0) = T_w = T_\infty + b_1 x^2 \,(\text{PST case}), \\ &-kT_y(x,0) = b_2 x^2 \,(\text{PHF case}), \\ &T(x,\infty) = T_\infty, \end{aligned} \tag{2.2}$$

where $d = 1$ denotes stretching and $d = -1$ denotes shrinking sheets, respectively, c is a positive constant measuring the rate of stretching or shrinking, ν is the kinematic viscosity, ρ is the density and η_0 is the material constant for the couple stress fluid. Two kinds of general heating processes, namely, the prescribed surface temperature (PST) and the prescribed wall heat flux (PHF) are accounted for.

3. Exact solutions

3.1. Solution of the flow field

Taking into consideration the below similarity transformations (see [2,26])

$$\eta = y\sqrt{\frac{c}{\nu}}, \quad u = cxf'(\eta), \quad v = -\sqrt{c\nu}f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \tag{3.3}$$

leading to axial wall constraint $v_w = -\sqrt{c\nu}f(0)$, the governing equations of motion and boundary conditions (2.1) and (2.2) are reduced to the similarity form

$$\begin{aligned} &-Cf^{(5)} + f''' + ff'' - f'^2 - Mf' = 0, \\ &\theta'' + Pr(f\theta' - 2f'\theta) = 0, \end{aligned} \tag{3.4}$$

$$\begin{aligned} &f(0) = s, \quad f'(0) = d, \quad f'(\infty) = 0, \quad f''(\infty) = 0, \quad f'''(\infty) = 0, \\ &\theta(0) = 1\,(\text{PST case}), \quad \theta'(0) = -1\,(\text{PHF case}), \quad \theta(\infty) = 0. \end{aligned} \tag{3.5}$$

Here $C = \frac{c\eta_0}{\rho\nu^2}$ is the couple stress parameter and $M = \frac{\sigma B_0^2}{c\rho}$ is the magnetic interaction strength parameter. In the absence of the couple stress parameter C, the fluid is just the classical nonpolar regular and Newtonian. In this case, solution is known to be unique for the stretching sheet problem, see [8,27], and further, only dual solutions exist for the shrinking sheet problem for some combinations of suction and magnetic field parameters, see [28,29]. Triple solutions were recently encountered for the shrinking sheet problem in the recent publication [30], whenever the viscoelasticity is taken into account.

Following the above citations, physical solutions for both stretching and shrinking sheets come from the exponential relation

$$f(\eta) = s + d\frac{1 - e^{-\lambda\eta}}{\lambda}. \tag{3.6}$$

Substituting (3.6) into the first of (3.4) gives the following fourth order algebraic equation for the characteristic parameter λ

$$d + M + \lambda(s - \lambda + C\lambda^3) = 0. \tag{3.7}$$

The corresponding four roots of polynomial equation (3.7) are exactly expressed, respectively by

$$\begin{aligned} &\lambda = \frac{1}{2}\left(\sqrt{\lambda_3} - \sqrt{\frac{2 - \frac{2s}{\sqrt{\lambda_3}} - C\lambda_3}{C}}\right), \\ &\lambda = \frac{1}{2}\left(\sqrt{\lambda_3} + \sqrt{\frac{2 - \frac{2s}{\sqrt{\lambda_3}} - C\lambda_3}{C}}\right), \\ &\lambda = -\frac{1}{2}\left(\sqrt{\lambda_3} + \sqrt{\frac{2 + \frac{2s}{\sqrt{\lambda_3}} - C\lambda_3}{C}}\right), \\ &\lambda = \frac{1}{2}\left(-\sqrt{\lambda_3} + \sqrt{\frac{2 + \frac{2s}{\sqrt{\lambda_3}} - C\lambda_3}{C}}\right), \end{aligned} \tag{3.8}$$

with the dummy variables

$$\begin{aligned} \lambda_1 &= -2 + 72C(d+M) + 27Cs^2 \\ &\quad + \sqrt{-4\lambda_2^3 + \left(-2 + 72C(d+M) + 27Cs^2\right)^2}, \\ \lambda_2 &= 1 + 12C(d+M), \\ \lambda_3 &= \frac{2}{3C} + \frac{\lambda_1^{1/3}}{32^{1/3}C} + \frac{2^{1/3}\lambda_2}{3C\lambda_1^{1/3}}. \end{aligned}$$

It is worth noting that the solutions given in Eq. (3.8) may or may not exist, depending on the values of the physical parameters involved, since the square roots may give complex values. However, if $\lambda_3 < 0$, then all solutions are imaginary, so no solutions (of the prescribed form (3.6)) to the flow problem exist in that case. Otherwise one or more solutions could be possible.

Having chosen the appropriate roots (with a positive sign) from (3.8), the velocity profile for both stretching and shrinking surfaces is determined after differentiating (3.6)

$$f'(\eta) = de^{-\eta\lambda}, \tag{3.9}$$

and the skin friction coefficient of physical significance $-f''(0)$ is explicitly given by

$$-f''(0) = d\lambda. \tag{3.10}$$

3.2. Solution of the temperature field

Having determined f in (3.6), by means of the intermediate variable

$$t = -e^{-\lambda\eta},$$