International Journal of Heat and Mass Transfer 72 (2014) 9–22

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Mixed convection of a heated rotating cylinder in a square enclosure

Chuan-Chieh Liao, Chao-An Lin *

Department of Power Mechanical Engineering, National Tsing Hua University, Hsinchu 30013, Taiwan

ARTICLE INFO

Article history:
Received 27 August 2013
Received in revised form 30 December 2013
Accepted 30 December 2013
Available online 23 January 2014

Keywords:
Rotating cylinder
Natural and mixed convection
Immersed boundary method

ABSTRACT

Numerical investigations were carried out for natural and mixed convection within domains with stationary and rotating cylinder by using an immersed-boundary method. The method was first validated with flows induced by natural convection in the annulus between concentric circular cylinder and square enclosure. Steady mixed convection in a square enclosure with an active rotating cylinder was further investigated for different rotating speeds. The parameters investigated in the study included Rayleigh number, Prandtl number and the aspect ratio between inner cylinder and outer enclosure. The heat transfer quantities of the system were obtained for different Rayleigh numbers (*Ra*) within the range of 10^4–10^6. The influence of rotation on the instability of the flow at different Prandtl numbers within the range of 0.07–7.0 was also investigated. Local and average heat transfer characteristics were fully studied around the surfaces of both inner cylinder and outer enclosure.

1. Introduction

Natural convection in an enclosure has been frequently investigated due to its wide application in nuclear reactors, heat exchangers, electronic packaging, and so on. Natural convection in rectangular and circular enclosures without inner object were reviewed by Ostrach [1] and other geometric configurations were further studied [2–4]. Another area of interest is the influence of immersed object on natural convection within an enclosure. These can be heat transfer within the annulus of horizontally concentric circular cylinders [5] or the natural convection in concentric annulus between an inner circular object and an outer rectangular enclosure [6]. The latter geometry is commonly investigated, and a large number of numerical [7–12] and experimental [13–15] studies had been performed to investigate natural convection within rectangular enclosure with varying aspect ratio, eccentricity and cross-section.

On the other hand, mixed convection induced by the rotation of the cylinder within rectangular enclosure was also investigated. Ghaddar and Thiele [16] presented results of a constant heat flux rotating cylinder within an isothermal rectangular enclosure using spectral element method. The rotation was found to increase the heat transfer at low Rayleigh number, and on the contrary, noticeable decrease of the heat transfer was observed at high Rayleigh number. Other studies focus on the interaction of the rotating cylinder with the natural convection induced by the differentially heated enclosure walls. For example, Fu et al. [17] found that the cylinder rotating direction had an important effect in enhancing natural convection heat transfer within the enclosure. Costa and Raimundo [18] investigated the influence of an active rotating cylinder with a free boundary condition. The size of the cylinder was found to have a significant influence on the resulting heat transfer. Hussain and Hussein [19] analyzed a conductive rotating cylinder at different locations within the square enclosure for fixed Prandtl number. Also, the influence of rotating cylinder on heat transfer with nanofluids was further studied by Roslan et al. [20].

Most of the aforementioned studies investigated the influence of Rayleigh number and rotational Reynolds number on the mixed convection, but few of them addressed the effect of Prandtl number and is investigated here. It is expected that the flow may experience instability at low Prandtl number. Another focus is the size of the cylinder on the heat transfer characteristics. The geometric setup consists of a rotating cylinder placed within an isothermal enclosure. Here, the rotating cylinder is mimicked by the immersed boundary method [21,22] within the Cartesian coordinate.

The remainder of this paper is organized as follows. The present methodology is first validated with flows induced by natural convection in the annulus between concentric circular cylinder and square enclosure. Mixed convection in a square enclosure with an isothermally rotating circular cylinder is further investigated and the parameters investigated include Rayleigh number, aspect ratio and Prandtl number for different rotating speeds. Finally, a brief summary is given according to the above results.

2. Methodology of the immersed-boundary technique

2.1. Mathematical formulation

The schematic configuration of the system considered in the present study is shown in Fig. 1. Consider the problem of a viscous incompressible fluid with thermal convection in a two-dimen-

* Corresponding author. Tel.: +886 3 5742602; fax: +886 3 5722840.
E-mail address: calin@pme.nthu.edu.tw (C.-A. Lin).

0017-9310/$ - see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.12.081

Nomenclature

D	diameter of the cylinder
$\boldsymbol{f}_M$	momentum forcing
$\boldsymbol{f}_E$	energy forcing
$\boldsymbol{g}$	gravitational acceleration
h	heat transfer coefficient
k	thermal conductivity
L	height of the cavity
n	normal direction along the boundary
Nu_{Local}	local Nusselt number
Nu_{Mean}	surface-averaged Nusselt number
p	pressure
Pr	Prandtl number = $\frac{\nu}{\alpha}$
R	radius of the cylinder
Ra	Rayleigh number = $\frac{g\cdot\beta\cdot(T_s-T_0)\cdot L^3}{\nu\cdot\alpha}$
Re	Reynolds number = $\frac{V_{max}D}{\nu}$
Ri	Richardson number = $\frac{Ra}{Pr\cdot Re^2}$
S	perimeter of the circular cylinder
t	time
T	temperature
u, v	velocity components in x and y directions
V_{max}	maximum velocity of the rotating cylinder = $R\omega$
x, y	Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ρ	density
ν	kinematic viscosity
θ	angular coordinate
ω	angular velocity of cylinder
χ	characteristic function for points of the solid

sional rectangular domain containing an immersed massless boundary, where no-slip and isothermal boundary conditions are imposed on both boundaries.

The governing equations of this fluid-structure interaction system with thermal condition are

$$\nabla\cdot\boldsymbol{u}=0. \tag{1}$$

$$\frac{\partial\boldsymbol{u}}{\partial t}+\nabla\cdot(\boldsymbol{u}u)=-\nabla p+\frac{1}{Re}\Delta\boldsymbol{u}+\frac{Ra}{Re^2Pr}T+\boldsymbol{f}_M, \tag{2}$$

$$\frac{\partial T}{\partial t}+\nabla\cdot(\boldsymbol{u}T)=\frac{1}{RePr}\Delta T+\boldsymbol{f}_E. \tag{3}$$

Here, $\boldsymbol{x}=(x,y), \boldsymbol{u}(\boldsymbol{x},t)$ is the fluid velocity with components u and v, and $p(\boldsymbol{x},t)$ is the fluid pressure. ∇ and Δ are the usual Cartesian-coordinate gradient and Laplacian. Ra is the Rayleigh number, which is defined as $Ra=\frac{g\cdot\beta\cdot(T_s-T_0)\cdot L^3}{\nu\cdot\alpha}$ with the gravitational acceleration $\boldsymbol{g}$, the thermal expansion coefficient β, the temperatures of the immersed object T_s and the outer enclosure T_0, characteristic length L, kinematic viscosity ν and thermal diffusivity α. Pr is the Prandtl number, which is defined as $Pr=\frac{\nu}{\alpha}$. Re is the Reynolds number, which is defined as $Re=\frac{(R\omega)D}{\alpha}$ with angular velocity ω, radius R and diameter D of the rotating cylinder. It is noted that the Cartesian coordinates $\boldsymbol{x}$, time t, velocity $\boldsymbol{u}(\boldsymbol{x},t)$, temperature T and pressure $p(\boldsymbol{x},t)$ are normalized with $L, L/R\omega, R\omega, (T_s-T_0)$ and $\rho(R\omega)^2$, respectively [19].

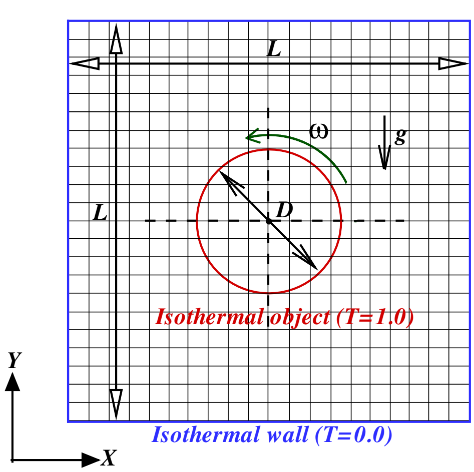

Fig. 1. Computational domain and boundary conditions for mixed convection in the annulus between concentric rotating circular cylinder and square enclosure.

Note that the discrete momentum forcing $\boldsymbol{f}_M(\boldsymbol{x},t)$ is applied to satisfy the no-slip condition on the immersed boundaries, as in [23]. Also, the discrete energy forcing $\boldsymbol{f}_E(\boldsymbol{x},t)$ is used to satisfy the prescribed thermal conditions on immersed boundaries. Both momentum forcing and energy forcing are applied only at the nodes adjacent to the immersed boundary in an Eulerian grid system.

2.2. Numerical scheme

The numerical procedure used herein consists of a finite-volume method discretized in Cartesian coordinates with a staggered-grid arrangement of dependent variables. This procedure is based on the integration of the transport equations over arbitrary control volumes, leading to the conservation of mass and the balance of momentum, energy and any scalar flow property over

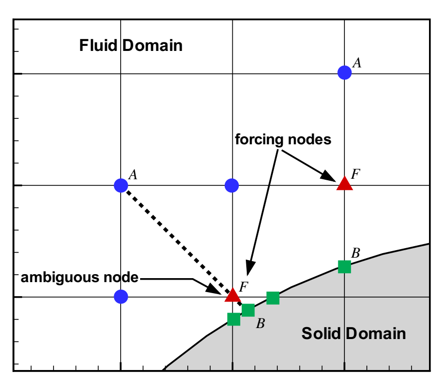

Fig. 2. Schematic diagram for the interpolation procedure.

Table 1
Comparisons of the simulated surface-averaged Nusselt number for different uniform grids with those of Moukalled and Acharya [6] and Shu et al. [8] for grid-function convergence tests.

	200 × 200	300 × 300	400 × 400	600 × 600	Ref. [6]	Ref. [8]
Nu_{Mean}	12.055	12.189	12.194	12.193	12.214	12.212