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Filter solution of inverse heat conduction problem using measured temperature history as remote boundary condition

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ABSTRACT

The inverse heat conduction problem (IHCP) involves estimation of a surface heat flux from transient temperature measurements inside a heat conducting body. Commonly an insulated remote boundary or one with a known heat transfer coefficient is modeled. However, in many practical applications, the precise thermal condition at the remote boundary is not known. In this paper, a method of accounting for thermal action at the remote boundary using a second measured temperature history is presented. The measurement need not be at an actual boundary but can be at an interior point in the domain.

The IHCP solution herein is achieved through the filter coefficient method, which uses filter coefficients in a convolution summation. By using the filter technique, near real-time heat flux measurements can be continuously obtained in manufacturing settings to enhance productivity. Also the filter concept opens the way for the development of new scientific instruments that incorporate inverse problem methods.

1. Introduction

The inverse heat conduction problem (IHCP) is typically defined as the problem of using internal temperature measurements to find heat fluxes on the surface. Sometimes this specific task is termed a *boundary IHCP* to emphasize recovery of the unknown surface heating action. The IHCP has been studied widely during last few decades, and notable texts have been authored by Beck et al. [1], Alifanov [2], Ozisik and Orlande [3] and Murio [4], to name a few. Several different methods have been developed and demonstrated for solving IHCPs. All boundary IHCP solutions assume full knowledge of the governing equation and thermal material properties, and full knowledge of boundary conditions on all but the active surface under investigation.

Remote boundaries are typically assumed to be insulated or cooled with a known heat transfer coefficient. For example, Ijaz et al. [5] consider estimation of heat flux histories on two faces of a two-dimensional domain. Their model assumes perfectly insulated surfaces on the other two faces, and temperature data are gathered on these insulated surfaces to drive the solution of the IHCP. Chen, et al. [6] also consider a two-dimensional problem, with two faces insulated, but the third face at a homogeneous temperature condition. In their method, the temperature, and not the heat flux, is determined on the fourth face of the domain. Mulcahy et al. [7] consider a problem in an annular three-dimensional geometry and seek the heat flux history on the inner surface of the annulus as a function of angle θ and axial coordinate z. They assume that all the other surfaces are insulated. Chan [8] performs an interesting study to determine the heat flux on a cylinder cooled by a jet. In this two-dimensional problem, measured temperatures on the surface of the cylinder are used to compute the heat flux on the cylinder, but this is done through solution of the flow field around the cylinder. In this case, insulated and no-slip boundaries are imposed on the bounding walls of the domain. Khajehpour et al. [9] tackle two-dimensional problems with finite element analysis and obtain solutions by splitting the domain into sub-regions. However, they do assume the remote boundaries are either insulated or have known convection cooling.

Chen, et al. [10] consider a slightly different problem. Their analysis of a one-dimensional domain considers two subsurface sensors, but seeks to compute the heat flux at both exposed surfaces.

Since IHCPs are mathematically ill-posed, an appropriate regularization method needs to be applied in order to convert the original ill-posed problem to a nearby well-posed problem and achieve a solution for it. Tikhonov regularization (TR) [11,12] is a common technique to stabilize the IHCP. It is commonly applied over the whole time domain which means that the solution procedure needs to access all the observations at once [1].

More recently, efforts have been made to develop real-time or filter forms for processing temperature data to solve the IHCP. Khorrami, et al. [13] employ an interesting approach based on the premise that the heat flux component at a particular time is

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Nomenclature

C	volumetric specific heat, J/m^3-K
f	filter coefficients (X21 case)
$\mathbf{F}$	filter matrix (X21 case)
g	filter coefficients (X12 case)
G	Green's function
$\mathbf{G}$	filter matrix (X12 case)
k	thermal conductivity, W/m-K
L	location of temperature boundary condition, Eq. (3), m
m_f	number of future time steps
m_p	number of past time steps
q	heat flux, W/m^2
S	sum of squares of the temperature error, K^2
t	time, s
T	temperature, K
x	spatial coordinate, m
x'	dummy integration variable, Eq. (6)
$\mathbf{X}$	sensitivity matrix for unknown surface heat flux
y	measured temperature at boundary $x = L$
Y	measured temperature at location $x = x_1$
$\mathbf{Z}$	sensitivity matrix for measured temperature boundary condition at $x = L$

Greek/Roman

α	thermal diffusivity, k/C, m^2/s
α_T	Tikhonov regularization parameter
σ	standard deviation
σ_2	variance
β	eigenvalue
ϕ	step response function for unit heat flux at $x = 0$
η	step response function for unit temperature at $x = L$
τ	integration variable, Eq. (6)

Subscripts

0	surface location or reference value
1	location of measurement sensor
i	time index
m	eigenvalue index
M	time index; any time, or the current time
ss	steady state
N	last time index
X21	Cartesian heat conduction problem with type 2 and type 1 boundary conditions

Superscript

$\sim$	dimensionless parameter

a linear combination of the temperature around the time of its occurrence. They use an artificial neural network to determine the required coefficients of the linear filter based on detailed numerical simulation of the target process. Deng and Hwang [14] also use artificial neural networks to generate the filter solutions for heat flux estimation based on temperature histories. Ijaz et al. [5] solve a transient IHCP by using Kalman filter. LeBreux et al. [15] use a combination of Kalman filtering and recursive least squares to achieve a real-time algorithm to determine heat flux history for industrial applications. Lamm [16,17] employs a sequential computation based on TR by using a recent subset of available data and a limited number of past predictions and observations. Cabeza, et al. [18] studied the filter effect of the function specification method and the truncated singular value decomposition method in a sequential form [19]. By examining the power spectral densities of the two methods, they concluded that the regularization methods act as band-pass filters. Most recently, Woodbury and Beck [20] study the structure of the TR problem and conclude that the method can be interpreted as a sequential filter formulation for continuous processing of data. They show that the computed heat fluxes using the whole domain solution and the filter coefficient solution are virtually the same for the constant-property solutions.

The filter method [1,20] used in this paper is a representation of one of many IHCP solution methods, such as Tikhonov Regularization, in a digital filter form. It is not a new or different method of solving IHCPs, but merely a representation of the solution in a form suitable for continuous (online) processing of data.

Measuring heat flux has great significance in several scientific experiments as well as industrial applications such as monitoring manufacturing processes and fire safety tests. Digital filter solutions are especially advantageous for developing new instruments for near real-time heat flux estimation from continuous temperature histories.

In this paper, a method is developed to incorporate the temperature measurement history from a second subsurface sensor as a remote boundary condition in an IHCP solution. The second measurement may, or may not, coincide with the physical boundary of the domain. Tikhonov regularization is used to stabilize the solution and the resulting algorithm is written in filter form [20]. An example problem is considered and both the whole domain method and the sequential filter solution method are illustrated. The filter solution of the IHCP has a number of advantages including simplicity, continuous operation and application to moderate nonlinearity manifested by temperature-dependent thermal properties [21] which makes it an appropriate approach for near real-time heat flux estimation.

2. Problem description

The IHCP solution typically starts by minimizing a sum of squares function, which is the sum of the squares of the difference between the measured and calculated temperatures at a location x_1. The calculated temperatures are functions of the unknown surface heat flux. Some form of regularization is needed, and the function specification method [1] or Tikhonov regularization (TR) [11,12] are only two of many techniques. For the function specification method, the heat flux is found using a sum of squares function that uses both past and future information. The Tikhonov whole domain regularization method will have a sum of squares function over the total time domain. However, the nature of the IHCP is such that a given heat flux component is a linear function of only a limited number of past and future measured temperatures. This point is illustrated below.

For simplicity, at this point a sum of squares function over the total time domain will be used in this analysis. Furthermore this time domain is large enough so that there is a long middle time region in which the very early conditions and the final time conditions have negligible effect upon the heat flux. This, too, is discussed further below.

The describing heat conduction equation is

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = C\frac{\partial T}{\partial t}, \quad 0 < x < L,\ t > 0 \qquad (1)$$