International Journal of Heat and Mass Transfer 72 (2014) 299–307

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A method for inverse analysis of laser surface heating with experimental data

Sen-Yung Lee *, Tsung-Wen Huang [1]

Department of Mechanical Engineering, National Cheng Kung University, Tainan 701, Taiwan, Republic of China

ARTICLE INFO

Article history:
Received 12 June 2013
Received in revised form 15 December 2013
Accepted 26 December 2013
Available online 1 February 2014

Keywords:
Laser surface heating
Inverse heat conduction
Time-dependent boundary conditions

ABSTRACT

This paper proposes a solution method for inverse laser surface heating problem. By minimizing the mean square error between the experimental data obtained from inside the body and the estimated data from the derived analytical solution of a laser surface heating problem with time-dependent boundary conditions, the temperature function at the laser heating end can be determined. Consequently, the temperature distribution and the heat flux over the entire time and space domains can also be obtained. In addition, the integral transform and tedious numerical operations are not required in the proposed solution method. Mathematical and experimental examples are given to illustrate the simplicity, efficiency, and accuracy of the proposed method.

1. Introduction

Inverse heat conduction problems (IHCPs) arise in many heat transfer situations when experimental difficulties are encountered in measuring or producing the appropriate boundary conditions. Practical applications are the estimation of the temperature and the heat flux at the surface of the body under investigation. Laser surface heating, heat exchangers, combustion chambers, and calorimeter-type instrumentation are typical examples.

The present study considers laser heat treatment on a surface. It is known that under the laser surface hardening process, the surface temperature must be maintained above the critical transformation temperature and below the melting point. Thus, during the heating process, accurate estimations of the temperature, heat flux, and surface absorptivity on the surface are important. Due to the difficulty of measuring the temperature and heat flux from the heated surface directly, these physical quantities are estimated from the measured temperature data inside the body within the heating time interval. Such estimation is a typical inverse heat conduction problem.

Many numerical techniques have been proposed for solving one-dimensional IHCPs. Among these methods, the finite difference method, the finite element method, and the boundary element method are the numerical tools of choice for the modeling and simulation of IHCPs. Lesnic and Elliott [1] employed Adomian's decomposition and the mollification method to deal with noisy input data and obtained a stable approximate solution. Monde and Mitsutake [2], Monde et al. [3] and Woodfield et al. [4] developed an analytical method using the Laplace transform and half polynomial series of time with a time lag to estimate thermal diffusivity, surface temperature, and heat flux for one-dimensional IHCPs. They recommended choosing the measurement points as close to the surface as possible to give a good estimation. Hon and Wei [5], Jin and Zheng [6], and Yan et al. [7] developed meshless and integration-free numerical schemes based on the use of the fundamental solution as a radial basis function for one-dimensional IHCPs. However, the resulting matrix equation is complicated and it is difficult to obtain accurate results. For the problem of laser heat treatment on a surface, an analysis includes the utilization of the inverse study by the conjugate gradient method, with temperatures measured near the heated surface was given by Wang et al. [12]. They show that there is a reasonable agreement on the surface temperature inversely obtained from the individual measurement of two different sensor locations. Chen and Wu [8] proposed a hybrid technique of the Laplace transform and the finite difference method in conjunction with experimental temperature data inside a test cylinder given by Wang et al., to predict the laser-heated surface temperature. In their numerical example with an exact solution, the error of the estimated heat flux is approximately 3%.

Most existing solution methods have to deal with tedious numerical problems, such as the inverse Laplace transform, stability in numerical schemes, and large numbers of cells or elements in matrix operations. The present study develops an integral-transform-free solution method for one-dimensional IHCPs with

* Corresponding author. Tel.: +886 6 2757575x62150.
E-mail addresses: sylee@mail.ncku.edu.tw (S.-Y. Lee), fir@mail.mirdc.org.tw (T.-W. Huang).
[1] Tel.: +886 7 3513121x2454.

0017-9310/$ - see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.12.067

Nomenclature

Bi_L	Biot number at the right end of cylinder
$B_n(\tau)$	dimensionless quantity defined in Eq. (31)
$B_{nj}(\tau)$	dimensionless quantity defined in Eq. (A12)
c	specific heat (W s/kg °C)
C_j	coefficients of dimensionless time-dependent function
$\mathbb{C}$	vector in matrix equation
$\bar{\mathrm{E}}$	function representing the error
$f(t)$	time-dependent temperature function at the heating end
$\bar{f}(\tau)$	dimensionless time-dependent function at the heating end
$f_0(t)$	time-dependent function for third-kind boundary condition
$\overline{f_0}(\tau)$	dimensionless time-dependent function for third-kind boundary condition
$\mathrm{F}(\xi,\tau)$	dimensionless quantity defined in Eq. (23)
$g_1(\xi)$, $g_2(\xi)$	shifting functions
h_L	convection coefficient at the right end of cylinder (W/m^2)
k	thermal conductivity (W/m °C)
k_L	conductivity coefficient at the right end of cylinder (W/m °C)
L	length of cylinder (m)
$q(\xi,\tau)$	dimensionless heat flux
$\mathbb{R}$	vector in matrix equation
T	temperature (°C)
T_r	reference temperature (°C)
$T_0(x)$	initial temperature (°C)
$T^{mea}(x_m, t_r)$	temperature measured at (x_m, t_r)
t	time variable (s)
t_r	time of temperature measurement
x	spatial-domain variable (m)
$\mathbb{Z}$	matrix in matrix equation

Greek symbols

α_n	dimensionless quantity defined in Eq. (33)
δ_n	norm of nth eigenfunctions
ε_n	dimensionless quantity defined in Eq. (A6)
$\phi_n(\xi)$	nth eigenfunction
$\gamma_n(\tau)$	dimensionless quantity defined in Eq. (32)
$\eta_j(\xi,\tau)$	dimensionless quantity defined in Eq. (A11)
$\eta_{jr}(\xi)$	dimensionless quantity defined as $\eta_{jr}(\xi) = \eta_j(\xi,\tau_r)$
κ_n	dimensionless quantity defined in Eq. (A7)
λ_n	nth eigenvalue
$v(\xi,\tau)$	transformed function
θ	dimensionless temperature
θ_0	dimensionless initial temperature
ρ	mass density (kg/m^3)
τ	dimensionless time variable
τ_r	dimensionless measured times
υ_n	dimensionless quantity defined in Eq. (A8)
ξ	dimensionless spatial-domain variable

time-dependent boundary conditions. The general analytic solution form of the heat conduction problem with time-dependent boundary conditions is derived by extending the methods of Lee and Lin [9], Lee et al. [10], and Chen et al. [11]. The measured experimental data obtained from inside the body are processed and approximated by a polynomial function.

By minimizing the mean square errors between the measured experimental data obtained from inside the body and the estimated data from the derived analytical solution form, the unknown temperature function at the laser heating end, in polynomial function form, can be determined. Consequently, the temperature distribution and the heat flux over the entire time and space domains can also be obtained. Mathematical and experimental examples are given to illustrate the analysis. The developed solution method is simple, efficient, and accurate. It can be applied to problems with various kinds of time-dependent boundary conditions.

2. Analytic solutions

Consider the laser heat treatment of the surface of a cylinder with constant material properties, as shown in Fig. 1. The cylinder is heated by a laser beam at one end. The time-dependent temperature function,$f(t)$, at the heating end is to be determined. A nonhomogeneous third-kind boundary condition is applied at the other end $f_0(t)$. Along the length, the cylinder is enclosed by a highly insulated material, and thus the surface is considered as an adiabatic surface. The ratio of the length to the diameter is large enough to ensure that the problem is one-dimensional. The governing differential equation and the boundary conditions of the system are

$$k\frac{\partial^2 T(x,t)}{\partial x^2} = \rho c\frac{\partial T(x,t)}{\partial t}, \quad 0 < x < L, \quad t > 0 \tag{1}$$

$$T(0,t) = f(t), \quad \text{at} \quad x = 0, t > 0 \tag{2}$$

$$k_L\frac{\partial T(L,t)}{\partial x} + h_L T(L,t) = f_0(t), \quad \text{at} \quad x = L, \quad t > 0 \tag{3}$$

and the initial condition is

$$T(x,0) = T_0(x). \quad 0 \leqq x \leqq L, \quad t = 0 \tag{4}$$

where, x is the spatial-domain variable,t is the time variable, $T(x,t)$ is the temperature over the entire domain, k is the thermal conductivity coefficient, ρ is the mass density, c is the specific heat, L is the length of the cylinder, and T_0 is the initial temperature. k_L and h_L are the thermal conductivity and the heat convection coefficients at $x = L$, respectively. When $k_L = 0$, the temperature at the boundary is time-dependent. Otherwise, when $h_L = 0$, the heat flux at the boundary is time-dependent. In the present study, a thermocouple is located at $x = x_m$ to record the temperature history, $T^{mea}(x_m,t_r)$. Here, $t_r, r = 1 \sim p$, are the times of measurement within the laser heating time interval.

In terms of the following dimensionless quantities

$$\xi = \frac{x}{L}, \quad \theta_0 = \frac{T}{T_r}, \quad \theta_0 = \frac{T_0}{T_r}, \quad \tau = \frac{kt}{\rho c L^2}, \quad Bi_L = \frac{h_L L}{k_L},$$
$$\bar{f}(\tau) = \frac{f(t)}{T_r}, \quad \overline{f_0}(\tau) = \frac{f_0(t)L}{k_L T_r} \tag{5}$$

where Bi_L is the Biot number and T_r is the reference temperature, the boundary value problem becomes

$$\frac{\partial^2\theta(\xi,\tau)}{\partial\xi^2} = \frac{\partial\theta(\xi,\tau)}{\partial\tau}, \quad 0 < \xi < 1, \quad \tau > 0 \tag{6}$$

$$\theta(0,\tau) = \bar{f}(\tau), \quad \text{at} \quad \xi = 0, \quad \tau > 0 \tag{7}$$

$$\frac{\partial\theta(1,\tau)}{\partial\xi} + Bi_L\theta(1,\tau) = \overline{f_0}(\tau), \quad \text{at} \quad \xi = 1, \quad \tau > 0 \tag{8}$$

$$\theta(\xi,0) = \theta_0(\xi). \quad 0 \leqq \xi \leqq 1, \quad \tau = 0 \tag{9}$$

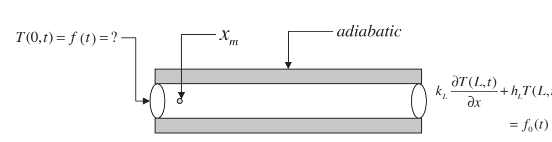

Fig. 1. Schematic diagram of the inverse problem.