International Journal of Heat and Mass Transfer 72 (2014) 352–361

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Estimation of the local heat-transfer coefficient in the laminar flow regime in coiled tubes by the Tikhonov regularisation method

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ARTICLE INFO

Article history:
Received 31 July 2013
Received in revised form 24 December 2013
Accepted 7 January 2014
Available online 3 February 2014

Keywords:
Heat-transfer enhancement
Coiled tubes
Local convective heat-flux estimation
Inverse heat conduction problem

ABSTRACT

Wall curvature is a widely used technique to passively enhance convective heat transfer that has proven to also be effective in the thermal processing of highly viscous fluids. These geometries produce a highly uneven convective heat-flux distribution at the wall along the circumferential coordinate, thus affecting the performance of the fluid thermal treatment. Although many authors have investigated the forced convective heat transfer in coiled tubes, most of them have presented the results only in terms of the Nusselt number averaged along the wall circumference. A procedure to estimate the local convective wall heat flux in coiled tubes is presented and tested in this paper: the temperature distribution maps on the external coil wall were employed as input data of the inverse heat conduction problem in the wall under a solution approach based on the Tikhonov regularisation method with the support of the fixed-point iteration technique to determine a proper regularisation parameter. The investigation was focused on the laminar flow regime.

1. Introduction

Wall curvature is among the most frequently used passive techniques to enhance convective heat transfer. The effectiveness of wall curvature occurs because it gives origin to the centrifugal force in the fluid: this phenomenon induces local maxima in the velocity distribution that locally increase the temperature gradients at the wall by maximising the heat transfer [1–6]. The asymmetrical distribution of the velocity field over the cross-section of the tube leads to a significant variation in the convective heat-transfer coefficient along the circumferential angular coordinate: it presents higher values at the outer bend side of the wall surface than at the inner bend side.

This irregular distribution may be critical in some industrial applications, such as in those that involve a thermal process. For instance, in food pasteurisation, the irregular temperature field induced by the wall curvature could reduce the bacteria heat-killing or could locally overheat the product. Therefore, to predict the overall performance of heat-transfer apparatuses that involve the use of curved tubes, it is necessary to know the local distribution of the convective heat-transfer coefficient not only along the axis of the heat-transfer section but also at the fluid-wall interface along the cross-section circumference.

Although many authors have investigated the forced convective heat transfer in coiled tubes, most of them have presented the results only in terms of the Nusselt number averaged along the wall circumference: only a few authors have studied the phenomenon locally, and most of them have adopted the numerical approach.

Yang et al. [7] presented a numerical investigation on the fully developed laminar convective heat transfer in a helicoidal pipe, with particular attention to the effects of torsion on the local heat-transfer coefficient. In particular, the authors reported the Nusselt number distribution varying the coil pitch, and they showed that, due to torsion, the local heat-transfer coefficient, compared to the case of an ideal torus, is increased on half of the tube wall while it is decreased on the other half.

Jayakumar et al. [8] numerically analysed the turbulent heat transfer in helically coiled tubes and presented the local Nusselt number at various cross sections along the curvilinear coordinate. The results showed that, on any cross section, the highest Nusselt number is on the outer side of the coil, and the lowest one is expected on the inner side. Moreover, the authors proposed a correlation for predicting the

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.01.019

Nomenclature

a	coil diameter (m)
D_{int}	tube internal diameter (m)
De	Dean number (–)
h	convective heat-transfer coefficient (W/m^2 K)
g	gravitational acceleration (m/s^2)
Gr	Grashof number $Gr = g \cdot \beta_f \cdot (\overline{T_w} - T_b) \cdot D_{\text{int}}^3 \cdot \rho_f^2/\mu_f^2$ (–)
Nu	Nusselt number (–)
q	convective heat flux (W/m^2)
$\mathbf{q}$	convective heat-flux vector (W/m^2)
q_g	internal heat generation per unit volume (W/m^3)
r	radial coordinate m
Re	Reynolds Number ($Re = w \cdot D_i \cdot \rho_f / \mu_f$) (–)
R_{env}	overall heat-transfer resistance between the tube wall and the surrounding environment (m^2 K/W)
T	temperature (K)
w	mean axial velocity
$\mathbf{X}$	sensitivity matrix (m^2 K/W)
P	coil pitch (m)
α	angular coordinate (rad)
β	volumetric thermal expansion coefficient (K^{-1})
Γ	dimensionless curvature of the coil ($\Gamma = D_{int}/a$)
Π	dimensionless torsion of the coil ($\Pi = P/\pi a$)
k	thermal conductivity (W/m K)
μ	dynamic viscosity (Pa s)
ρ	density (kg/m^3)

Subscripts, superscripts

b	bulk
env	environment
ext	external
f	fluid
int	internal

local Nusselt number as a function of the average Nusselt number and the angular location for both the constant temperature and the constant heat-flux boundary conditions.

Bai et al. [9] experimentally studied the turbulent heat transfer in helically coiled tubes using deionised water as the working fluid. As expected, they found that the local heat-transfer coefficient was not evenly distributed along the periphery of the cross section and that, in particular, at the outside surface of the coil, it was three or four times higher than that at the inside surface.

Bozzoli et al. [10] presented preliminary results regarding the local convective heat coefficient in coiled tubes for the laminar flow regime: because the estimation method proposed in this paper was based on the non-linear inverse heat conduction problem (IHCP), several strong approximations in the formulation of the model were adopted due to the restrictions imposed by the high computational cost of the minimisation algorithm.

To the authors' knowledge, the experimental data presented by Bozzoli et al. [10] are the only data available on the local heat transfer in coiled tubes for the laminar flow regime that is frequently encountered in industrial fields where highly viscous fluids are processed.

As shown by Bozzoli et al. [10], the solution of the IHCP in the wall, starting from the temperature distribution acquired on the external wall surface, is a robust tool to estimate the local convective heat-transfer coefficient on the interior wall surface. However, because IHCPs are generally ill-posed, the solution may not be unique and would have great sensitivity to small variations in the input data. To cope with this difficulty, many techniques have been proposed, and the most well-known are: function specification methods [11,12], iterative methods [13–15], methods based on filtering proprieties [16–18] and regularisation techniques[19,20].

Among the regularisation techniques, Tikhonov regularisation method [20] is perhaps the most common: it promotes the construction of stable approximate solutions to the original IHCP by solving a well-posed problem via the minimisation of an objective function. The objective function is expressed by the sum of the squared difference between the measured and the estimated temperature discrete data and of a regularisation parameter times a term that expresses the smoothness of the unknown quantity. The regularisation scheme suggested by Tikhonov and Arsenin [20] in the case of a particularly critical signal-to-noise ratio makes it possible to overcome the instability of the problem. The success of this approach relies on a proper choice of the regularisation parameter, and this is not an easy task.

The classical L-curve method to select a proper regularisation parameter, proposed by Hansen and O'Leary [21], was proven to produce good regularisation parameters in several cases; however, locating the corner in a robust way is not always an easy task because the L-curve sometimes displays several corners and sometimes the corner is not visible at all. On the contrary, the fixed-point method and its variants [22–24] have been proven to circumvent these difficulties on several test problems.

In the present paper, the Tikhonov regularisation method coupled to the fixed-point method for determining a proper value of the regularisation parameter was adopted to estimate the local convective heat flux at the fluid-wall interface in coiled tubes under the formulation of the linear IHCP in the wall. The temperature distributions on the external wall of the coiled tube, which are acquired using the infrared technique, were adopted as input data of the IHCP in the wall of the tube.

The investigation was particularly focused on the laminar flow regime, which is often found in coiled tube heat-exchanger applications. The purpose of this paper is twofold: to illustrate the estimation technique, which has been originally customised for this specific inverse problem, and to test it on an experimental case. The results, although obtained for a limited range of experimental conditions, are representative of a wide range of technical applications. Moreover, the data could be employed both as a useful benchmark for CFD results as well as in the design of coiled tube heat exchangers for the treatment of highly viscous fluids.

2. Experimental setup and data processing

In the present investigation, a smooth-wall helically coiled stainless steel type AISI 304 tube was tested. It was characterised by eight coils following an helical profile along the axis of the tube where the helix diameter and the pitch were of approximately 310 mm and 200 mm, respectively. The tube internal diameter was 14 mm, and the wall thickness measures 1.0 mm. This geometry yields a coiled pipe length L of approximately 10 m, a dimensionless curvature Γ of 0.045 and a torsion Π of 0.21.

The working fluid was conveyed by a volumetric pump to an holding tank, and it entered the coiled test section equipped with stainless-steel fin electrodes, which were connected to a power supply, type HP 6671A. This setup allowed investigation of the heat transfer performance of the tube under the prescribed condition of uniform heat flux generated by the Joule effect in the wall. The heat