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A note on micropolar fluid flow and heat transfer over a porous shrinking sheet

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ABSTRACT

The flow of micropolar fluid and heat transfer past a porous shrinking sheet is studied in this note. The main concern, unlike the recent numerical work of Bhattacharyya et al. (2012) [1], is to determine mathematically the bounds of multiple existing solutions of purely exponential kind. The presence of dual solutions are proved for the flow field, whose closed-form formulae are then derived. The energy equation is also treated analytically yielding exact solutions beneficial to understand the rate of heat transfer. Critical values for the existence or nonexistence of unique/multiple solutions are worked out. The exact form of velocity/temperature profiles and the skin friction/couple stress/heat transfer parameters enable one to easily catch the physical processes occurring in the present model.

1. Introduction

The model of microfluid first pioneered by Eringen [2] has been the focus of past research to explain the character of certain real fluid flows. This mathematical model takes into account a class of fluids having certain microscopic characters arising from the local structure and micromotions of the fluid elements [1]. Micropolar fluids are known as part of microfluids, which are also due to Eringen [3].

A rich survey of the past and recent works done concerning the micropolar fluids and, their technological and industrial applications were recently presented in [1]. One can also refer to the relevant stretching or shrinking body studies by Fang and Zhang [4], Khan et al. [5,6] and Turkyilmazoglu [7,8], amongst many others.

In the recent paper of [1], the flow of micropolar fluid and heat transfer over a permeable shrinking sheet was studied by numerical means. The deriving force of the current work is, as opposed to the aforementioned numerical treatments, to mathematically explore the physical problem under consideration. Expressions of threshold values for the nonexistence of the solutions or the existence of dual solutions are derived in closed-form. Such analytical formulae are very useful, not only to interpret the flow and temperature fields but also to view behaviour of the skin friction coefficient, couple stress coefficient and Nusselt number, which play major role in industrial applications.

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2. Problem formulation

We consider a steady two-dimensional flow of micropolar fluid due to an impermeable shrinking sheet, whose shrinking speed is $u=-cx;\ c>0$. The temperature is assumed to vary from a constant wall value to a constant free stream temperature. Making use of the classical boundary layer approximation, the governing equations of motion for the micropolar fluid and heat transfer were successfully extracted in [1], refer to Eqs. (1)–(6) in [1]. For the sake of being concise, we only give the final similarity equations that govern the flow motion and heat transfer

$$\begin{aligned}&(1+K)f'''+ff''-f'^2+Kh'=0,\\&(1+K/2)h''+fh'-f'h-K(2h+f'')=0,\\&\theta''+Prf\theta'=0,\end{aligned}\tag{2.1}$$

subject to the boundary conditions

$$\begin{aligned}&f(\eta)=s,\quad f'(\eta)=-1,\quad h(\eta)=-mf''(\eta),\quad \theta(\eta)=1\quad \text{at}\quad \eta=0,\\&f'(\eta)\to 0,\quad h(\eta)\to 0,\quad \theta\to 0\quad \text{as}\quad \eta\to\infty,\end{aligned}\tag{2.2}$$

where η is a scaled boundary layer coordinate, $f'(\eta)$ is the similarity velocity component, $h(\eta)$ is the similarity microrotation or angular velocity, s is the mass flux velocity with $s<0$ for suction and $s>0$ for injection, K is the material parameter and m represents a measurement for the concentration of microelements, respectively. Moreover, θ is the scaled fluid temperature and Pr is the usual Prandtl number.

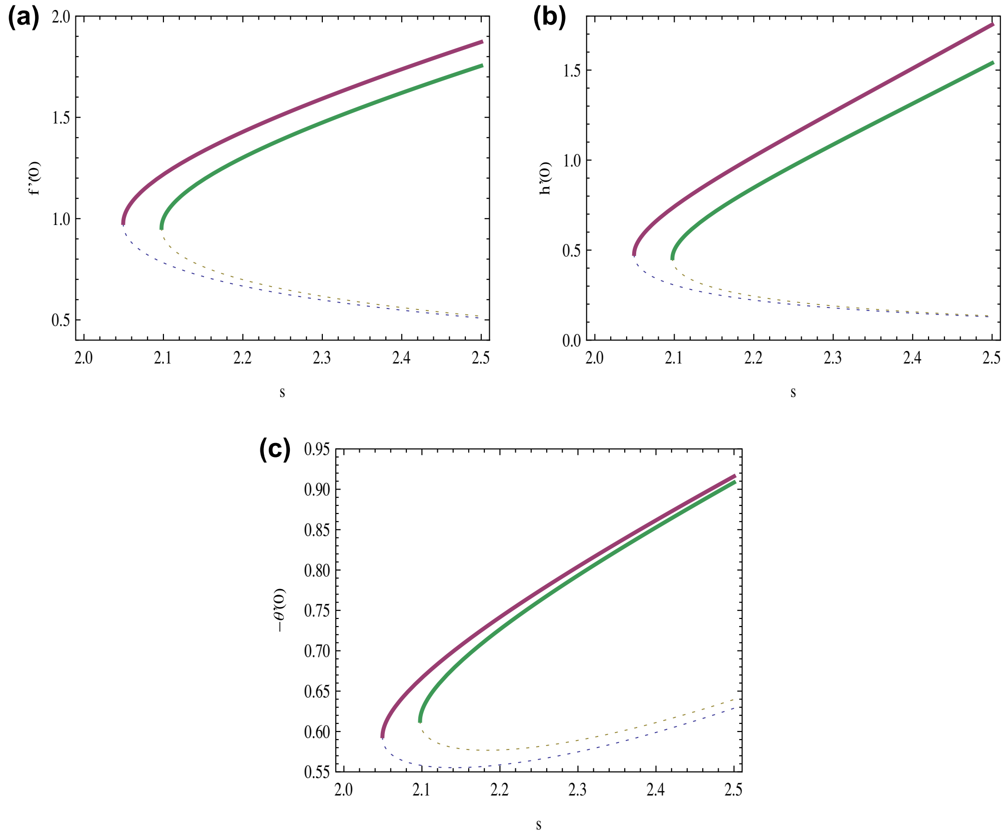

Fig. 1. The physical coefficients at $m = 1/2$ (thick curves for the first branch and dotted curves for the second ones, respectively, outer loop for $K = 0.1$ and inner loop for $K = 0.2$) as a function of s. (a) Skin friction coefficient $f''(0)$, (b) couple stress coefficient $h'(0)$ and (c) heat transfer coefficient $-\theta'(0)$ with $\mathrm{Pr} = 3/7$.

3. Exact solutions

A numerical treatment of Eqs. (2.1), (2.2) has already been given in [1]. We instead present exact analytic solutions in this section. Branch 1 or branch 2 refers to part of multiple solutions in the sequel.

3.1. Flow field

Based on the analytical solutions derived in Crane [9], Troy et al. [10], Mcleod and Rajagopal [11], Lawrence and Rao [12] and Pop and Na [13] for the stretching/shrinking sheet problems of particular type, it is desired to obtain exact solutions of the system (2.1)–(2.2), which is influenced by the material, concentration and wall suction parameters. The above literature enables us to assume a solution of the form

$$\begin{aligned} f(\eta) &= s - \frac{1 - e^{-\lambda\eta}}{\lambda}, \\ h(\eta) &= -mf''(\eta) = -m\lambda e^{-\eta\lambda}. \end{aligned} \tag{3.3}$$

Upon substitution, the entire boundary conditions in (2.2) are seen to be met by the solution (3.3). It is obvious that true solutions require the condition $\lambda > 0$. As a result, the momentum and angular velocity equations in (2.1) produce the relations

$$\begin{aligned} &-1 + s\lambda + (-1 + K(-1 + m))\lambda^2 = 0, \\ &2m(1 - s\lambda + \lambda^2) + K(2 + m(-4 + \lambda^2)) = 0, \end{aligned} \tag{3.4}$$

whose simultaneous solutions yield the dual solutions

$$\begin{aligned} &\left(m = 1/2, \quad \lambda = \frac{s - \sqrt{-4 - 2K + s^2}}{2 + K}\right), \\ &\left(m = 1/2, \quad \lambda = \frac{s + \sqrt{-4 - 2K + s^2}}{2 + K}\right), \\ &\left(m = \frac{8(1+K)^2}{\left(s - \sqrt{-4 + 4K + 8K^2 + s^2}\right)^2}, \quad \lambda = \sqrt{\frac{2}{m}}\right), \\ &\left(m = \frac{8(1+K)^2}{\left(s + \sqrt{-4 + 4K + 8K^2 + s^2}\right)^2}, \quad \lambda = \sqrt{\frac{2}{m}}\right). \end{aligned} \tag{3.5}$$

Thus, the first two of (3.5) are valid for $m = 1/2$ and the next two are for other values of m. It should be noticed that the structure of Eq. (3.5) puts restrictions on the solutions such that the first two holds for all physical $K \geqslant 0$ and

$$s \geqslant \sqrt{4 + 2K},$$

whereas the third for $0 \leqslant K \leqslant 1/2$ and

$$s \geqslant \sqrt{4 - 4K - 8K^2}$$

and the fourth, in addition to the above, for $K \geqslant 1/2$ and all s. Therefore, the critical values for the nonexistence or existence of dual solutions are given by the formulae