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Simulation of three dimensional double-diffusive throughflow in internally heated anisotropic porous media

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ABSTRACT

A model for double-diffusive convection in an anisotropic porous layer with a constant throughflow is explored, with penetrative convection being simulated via an internal heat source. The validity of both the linear instability and global nonlinear energy stability thresholds are tested using three dimensional simulation. Our results show that the linear threshold accurately predicts on the onset of instability in the steady state throughflow. However, the required time to arrive at the steady state increases significantly as the Rayleigh number tends to the linear threshold.

1. Introduction

Double-diffusive flows in porous media are widely encountered both in nature and in technological processes [1,2]. Bioremediation, where micro-organisms are introduced to change the chemical composition of contaminants is a very topical area, cf. Chen et al. [3], Suchomel et al. [4]. Contaminant/pollution transport is yet another area of multi-component flow in porous media which is of much interest in environmental engineering, cf. Curran and Allen [5], Ewing and Weekes [6], Franchi and Straughan [7]. Other very important and topical areas of salt/heat transport in porous flows are in oil reservoir simulation, e.g. Ludvigsen et al. [8], and salinization in desert-like areas, Gilman and Bear [9].

The literature on the study of the effect of vertical throughflow on convective instability in a porous medium is much less widespread, although recent studies include Shivakumara and Suma [10], Shivakumara and Khalili [12], Shivakumara and Sureshkumar [13], Nield and Kuznetsov [14], Hill [15], Hill et al. [16] and Capone et al. [17].

The effect of vertical throughflow on double-diffusive convection in a porous medium is important due to its applications in engineering (e.g. the directional solidification of concentrated alloys as well as in some energy storage devices) and geophysics (e.g. seabed hydrodynamics such as in hydrothermal vent systems). The difficulty in dealing with such instability problems is that one has to solve time dependent equations with variable coefficients, and the work in this direction is very limited. Shivakumara and Nanjundappa [18] used linear stability theory to analytically investigate the effects of quadratic drag and vertical throughflow on double diffusive convection in a horizontal porous layer using the Forchheimer-extended Darcy equation. Shivakumara and Sureshkumar [19] investigated the effects of quadratic drag and vertical throughflow on the linear stability of a doubly diffusive Oldroyd-B-fluid-saturated horizontal porous layer. Altawallbeh et al. [20] analytically studied using both linear and weakly nonlinear stability analyses the double-diffusive convection in an anisotropic porous layer heated and salted from below with an internal heat source and Soret effect. Shivakumara and Khalili [11] studied the problem of double-diffusive convection in a fluid filled anisotropic porous layer. Hill et al. [21] studied this problem but with the presence of an internal heat source to allow penetrative convection to occur. In this paper, we explore the model presented in Hill et al. [21] of double-diffusive throughflow in an internally heated anisotropic porous medium.

When the difference between the linear (which predicts instability) and nonlinear (which predicts stability) thresholds is very large, the validity of the linear instability threshold to capture the onset of the instability is unclear. Thus, we utilise the stability analysis of Hill et al. [21] to select regions of large subcritical instabiltiies and then develop a three dimensional simulation for the

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Nomenclature

$(x_1, x_2, x_3) = (x, y, z)$	Cartesian coordinates
$\mathbf{v}$	velocity
P	pressure
T	temperature
C	concentration of salt
$\mathbf{u}$	dimensionless velocity
p	dimensionless pressure
θ	dimensionless temperature
ϕ	dimensionless concentration of salt
μ	viscosity
ε	porosity
g	gravitational acceleration
κ_c	salt diffusivity
ρ	density
ρ_0	reference density
T_0	reference temperature
C_0	reference concentration
α_t	thermal expansion coefficient
α_c	solutal expansion coefficient
$K(z) = K_0 s(z)$	permeability of the porous medium
K_0	reference permeability
κ_t	effective thermal diffusivity of the porous medium
κ_s	thermal diffusivity of the solid component of the porous medium
κ_f	thermal diffusivity of the fluid component of the porous medium
c_p	specific heat of the fluid at constant pressure
c	specific heat of the solid at constant pressure
M	ratio of heat capacities
$Q\ (>0)$	internal heat source
$Ra_L = R_t^2$	thermal Rayleigh number
R_c^2	solute Rayleigh number
T_f	dimensionless form of the throughflow
a^2	horizontal wavenumber, $m^2 + n^2$
m, n	dimensionless disturbance wave vector
$\overrightarrow{\omega} = (\xi_1, \xi_2, \xi_3)$	vorticity vector
$\overrightarrow{\psi} = (\psi_1, \psi_2, \psi_3)$	potential vector
Lx	box dimension in the x direction
Ly	box dimension in the y direction

problem to test the validity of these thresholds. To achieve this we transform the problem into a velocity–vorticity formulation and utilise second order finite difference schemes. We use both implicit and explicit schemes to enforce the free divergence equation.

Standard indicial notation is used throughout the article, where $(x_1, x_2, x_3) = (x, y, z)$.

2. Mathematical formulation and governing equations

Utilising the approach of Hill et al. [21] (schematically shown in Fig. 1) let us consider a layer Ω of a water saturated porous medium bounded by two horizontal planes. Let $d > 0, \Omega = \mathbb{R}^2 \times (0, d)$ and $Oxyz$ be a cartesian frame of reference with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

Assuming that the Oberbeck–Boussinesq approximation is valid (cf. [22] and references therein), the flow in the porous medium is governed by Darcy's law

$$\frac{\mu}{K(z)} v_i = -P_{,i} - k_i g \rho(T, C), \tag{1}$$

$$v_{i,i} = 0, \tag{2}$$

$$\frac{1}{M} T_{,t} + v_i T_{,i} = \kappa_t \nabla^2 T + Q, \tag{3}$$

$$\varepsilon C_{,t} + v_i C_{,i} = \kappa_c \nabla^2 C, \tag{4}$$

where (2) is the incompressibility condition and (3) and (4) are the equations of energy and solute balance, respectively. The derivation of Eqs. (1)–(4) may be found in [23].

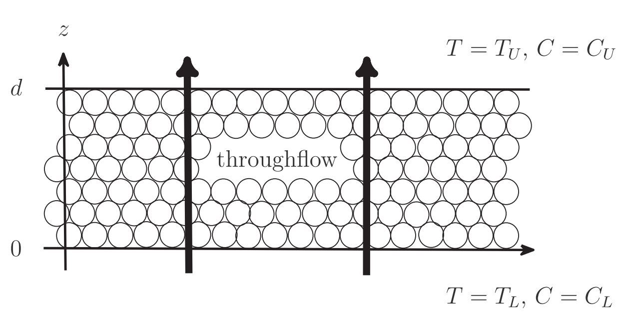

Fig. 1. Schematic representation of a cross-section of the system.

We have denoted $\mathbf{v}, P, T, C, \mu, \varepsilon, g$ and κ_c to be the velocity, pressure, temperature, concentration of salt, viscosity, porosity, gravitational acceleration and salt diffusivity, respectively. The density ρ is of the form

$$\rho(T, C) = \rho_0(1 - \alpha_t(T - T_0) + \alpha_c(C - C_0))$$

where ρ_0, T_0 and C_0 are a reference density, temperature and concentration, respectively, and α_t and α_c are the coefficients for thermal and solutal expansion, respectively.

The permeability of the porous medium is taken to be of the form

$$K(z) = K_0 s(z),$$

where K_0 is a reference permeability and $s(z) = 1 + \lambda_1 z/d$, with constant $\lambda_1 > -1$ to ensure $s(z) > 0$. The effective thermal conductivity of the saturated porous medium κ_t is defined by the ratio between the thermal diffusivity of the porous medium and the heat capacity per unit volume of the fluid:

$$\kappa_t = \frac{(1 - \varepsilon)\kappa_s + \varepsilon \kappa_f}{(\rho_0 c_p)_f}$$

where κ_s and κ_f are the thermal diffusivities of the solid and fluid components of the porous medium, respectively and c_p is the specific heat of the fluid at constant pressure. The coefficient M is the ratio of heat capacities defined by

$$M = \frac{(\rho_0 c_p)_f}{(\rho_0 c)_m}. \tag{5}$$

In (5) c is the specific heat of the solid, and

$$(\rho_0 c)_m = (1 - \varepsilon)(\rho_0 c)_s + \varepsilon(\rho_0 c_p)_f,$$

denotes the overall heat capacity per unit volume of the porous medium. The subscripts f, s and m referring to the fluid, solid and porous components of the medium, respectively.

The $Q\ (>0)$ term in (3) is a (constant) internal heat source, with its inclusion allowing the model to describe penetrative convection in the porous layer [24].

The temperature and concentration boundary conditions for the problem are $T = T_U$ and $C = C_U$ at $z = d$ and $T = T_L$ and $C = C_L$ at $z = 0$, where $C_L > C_U$, so that the system is being salted from below. We allow for the two cases of heating from below $T_L > T_U$ and from