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A comprehensive study of the effective thermal conductivity of living biological tissue with randomly distributed vascular trees

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ABSTRACT

The biological tissue can be treated as a porous medium consisting of randomly distributed vascular trees and solid tissue matrix. In this paper, taking into account the effects of geometric structures of vascular trees and blood flow, a fractal model for the effective thermal conductivity of living biological tissue is derived based on the assumptions that the mother channel diameters of vascular trees follow the fractal scaling law. The proposed model is expressed as a function of the thermal conductivities of solid tissue matrix and blood, structural parameters of vascular trees, porosity and properties of blood. It is found that the effective thermal conductivity of living biological tissue increases with the increase of branching levels m, length ratio α, decreases with the increase of diameter ratio β, and there exists a thermal conductivity ratio, at which the effective thermal conductivity is same for different porosities ε, below which the effective thermal conductivity increases with the increase of porosity ε, and above which the effective thermal conductivity decreases with the increase of porosity ε. A good agreement is obtained between the proposed model predictions and available experimental data for living tissue. The results show that blood flow plays an important role in increasing the effective thermal conductivity, and the proposed model with blood flow is more reasonable and can reveal more physical mechanisms of heat transfer in living biological tissue.

1. Introduction

The thermal conductivity of biological tissue (especially in vivo) is not only pivotal to deeply study heat transfer characteristics and mechanisms but also important in the clinical applications [1,2]. For example, in an effective hyperthermia treatment, in order to precisely control temperature distribution in therapy region and ensure that the tumor is mandated in the therapy temperature and the surrounding normal tissue is in safe temperature, an important aspect is the accurate knowledge of the thermal conductivity of tissue. Heat transport in biological tissue is usually expressed by bio-heat models, such as Pennes' [3] bio-heat transfer equation, Weinbaum and Jiji's [4] model, Wissler's [5] model, Baish's [6] model, etc., for establishment and validation of such bio-heat transfer models, the accurate values of thermal conductivity of biological tissue are also indispensable. Many researchers have attempted to accurately determine the thermal conductivity of biological tissue, e.g., the guarded hot plate technique proposed by Hill et al. [7], which is a steady-state method only used to measure the thermal conductivity of dead tissue. The step-temperature technique was developed by Valvano and Bowman et al. [1] and the pulse delay method was presented by Chen et al. [8], both of which are transient thermal techniques mainly to measure the thermal conductivity of living tissue. However, the data of thermal conductivity of biological tissue still are seriously insufficient due to the limitations of existing testing technology and the notable difference among the data reported in literatures. Thus, it is necessary to develop better and more accurate methods for determination of the effective thermal conductivity of biological tissue.

The biological tissue can be treated as a fluid saturated porous medium as the blood vessels can be considered as pores in which the blood permeates and the extra-vascular tissue can be considered as solid tissue matrix [9–11]. Some studies [12–16] showed that the blood vessels in tissue are often connected in the form of tree branching structure called vascular tree, which is statistically self-similar and fractal. For instance, Masters [16] demonstrated that the vascular tree in the normal human retina has statistically self-similar fractal structure, and the fractal dimension of the blood vessels is approximately 1.7. Buijs et al. [13] also studied the geometrical and fractal properties of the rat hepatic portal vein tree and found that the mean fractal dimension is 1.37. On the other hand, some experimental studies [1,17–19] showed that the blood flow via blood vessels has significant influence on the

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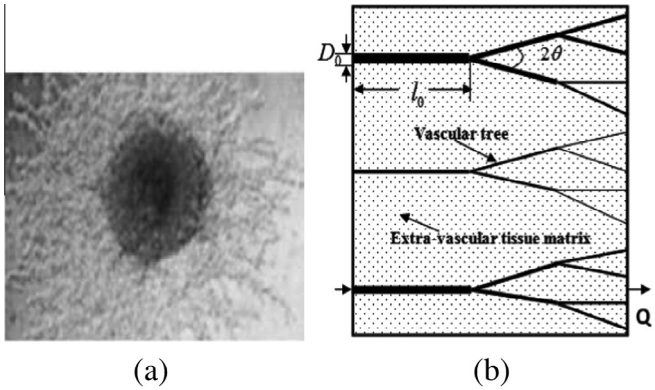

Fig. 1. (a) A typical brain tumor tissue with vascular trees around the brain tumor nucleus [15], and (b) schematic of the model of living biological tissue consisting of randomly distributed vascular trees surrounding solid tissue matrix.

thermal conductivity of living biological tissue. For example, Liang et al. [18] measured the thermal conductivity of a dead and a living snake and found that the thermal conductivity increases from 0.494 W/m K (without blood flow) to 0.58 W/m K (with blood flow). Thus, in this paper, we establish a new fractal model for the effective thermal conductivity of living biological tissue by considering the effects of geometric structures of vascular trees and blood flow.

Heat transfer properties (including heat conduction and heat convection) in symmetrical branching trees embedded in (or with no) matrix have been received much attention [20–27] recently. In this paper, we focus our attention on the derivation of a fractal model for the effective thermal conductivity of living biological tissue with randomly distributed vascular trees based on the constructal principle [20,21]. Figs. 1 (a) and (b), respectively show a typical brain tumor tissue with vascular trees around the brain tumor nucleus [15] and schematic of the model of living biological tissue consisting of randomly distributed vascular trees and surrounding solid tissue matrix. Mahjoob and Vafai [28] studied heat transfer in consecutive variable cross-sectional domains and found that the geometrical variations have a substantial impact on the heat transfer within the domain. The cross-sectional area in annular shape varies with the radius, so, for simplicity, the biological tissue with randomly distributed vascular trees is modeled as a rectangle shape as shown in Fig. 1 (b). The vascular trees are assumed to be symmetrically two-branching networks, i.e., every single vascular tree has the same length ratio α, diameter ratio β, branching angle θ, mother channel length l_0 and branching number $n = 2$ for simplification. But, the size distribution of the mother diameter D_0 follows the fractal scaling law.

This paper is organized as follows. Section 1 presents a brief introduction to the thermal conductivity of biological tissue, and after that Section 2 gives the characterization of biological tissue by the fractal theory. Then, in Section 3, a fractal model for the effective thermal conductivity of living biological tissue is derived by considering the effects of geometric structures of vascular trees and blood flow. The results and discussions of the proposed model are shown in Section 4, and finally the conclusions are given in Section 5.

2. Fractal characterization of biological tissue

As shown in Fig. 1(b), the biological tissue modeled as a rectangle shape consists of randomly distributed vascular trees and solid tissue matrix based on the constructal principle [20,21]. We assume that the diameter D_0 of mother channels of vascular trees follows the fractal scaling law. Thus, the cumulative number of such channels in the tissue can be expressed as follows [29–31]

$$N(L \geqslant D_0) = (D_{0\max}/D_0)^{D_f} \tag{1}$$

where L is the length scale, $D_{0\max}$ and D_f are the maximum diameter and the fractal dimension of the mother channels of vascular trees, respectively.Since there are numerous vascular trees, Eq. (1) can be considered as continuous and differential function. Then, differentiating Eq. (1) with respect to D_0 results in the number of channels whose diameters are within the infinitesimal range D_0 to $D_0 + dD_0$, i.e.,

$$-dN = D_f D_{0\max}^{D_f} D_0^{-(D_f+1)} dD \tag{2}$$

the negative sign in Eq. (2) implies that the vascular trees number decreases with the increase of channels size, and $-dN > 0$. The total number of vascular trees from the minimum mother diameter $D_{0\min}$ to the maximum mother diameter $D_{0\max}$ can be obtained from Eq. (1) as

$$N_t(L \geqslant D_{0\min}) = \left(\frac{D_{0\max}}{D_{0\min}}\right)^{D_f} \tag{3}$$

Dividing Eq. (2) by Eq. (3) obtains

$$\frac{-dN}{N_t} = D_f D_{0\min}^{D_f} D_0^{-(D_f+1)} dD_0 = f(D_0) dD_0 \tag{4}$$

where $f(D_0) = D_f D_{0\min}^{D_f} D_0^{-(D_f+1)}$ is the probability density function [29,32], and must satisfy the normalizing condition in fractal porous media. Yu et al. [29] proposed a criterion $\left(\frac{D_{0\min}}{D_{0\max}}\right)^{D_f} \cong 0$ must be satisfied for fractal analysis in a porous media.

3. Fractal model for the effective thermal conductivity of living biological tissue

Some experimental studies showed that the blood flow via blood vessels enhances the effective values of thermal conductivity of living biological tissue [1,17–19]. Thus in this section, a fractal model for the effective thermal conductivity of living biological tissue is derived by considering the synthetical effects of geometric structures of vascular trees and blood flow, and we assume that the effective thermal conductivity is contributed by heat conduction in vascular trees and solid tissue matrix as well as heat convection caused by blood flow.

The thermal conductivity by heat conduction in vascular trees and solid tissue matrix has been calculated by Li and Yu [33] and can be expressed as

$$\lambda_{cd} = \lambda_m(1-\varepsilon) + \lambda_b\left(1 + \alpha\frac{1-\alpha^m}{1-\alpha}\cos\theta\right)\frac{1-\alpha^{m+1}}{1-\alpha} \times \frac{1-n\beta^2\alpha}{1-(n\beta^2\alpha)^{m+1}}\frac{1-\alpha/n\beta^2}{1-(\alpha/n\beta^2)^{m+1}}\varepsilon \tag{5}$$

where λ_m is the thermal conductivity of solid tissue matrix (tissue), λ_b is the thermal conductivity of blood (the thickness of the vessel wall is ignored) and ε is the areal porosity (in two dimensions) of vascular trees.

Next, we focus our attention on the deduction of the thermal conductivity by heat convection caused by blood flow. The heat convection coefficient is defined by $h = (Nu \cdot \lambda_b)/D$. According to the assumptions by Chen and Cheng [22], the flow through each channel of the vascular trees is laminar and thermally fully developed, the Nusselt number remains constant at each level, so $h_k = h_0\beta^{-k}$. For a fully developed flow with constant heat flux in a uniform cross-section, we can gain a constant temperature difference ΔT between the blood vessels wall surface and bulk flow. Meanwhile the temperature difference ΔT for each level of the vascular trees is also assumed to be constant. Thus the heat convection caused by blood flow in a single vascular tree can be obtained as [22]