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Dual solutions of a mixed convection flow of nanofluids over a moving vertical plate

S.V. Subhashini *, R. Sumathi

Department of Mathematics, Anna University, Chennai 600 025, India

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ABSTRACT

This paper deals with the development of a mixed convection flow of nanofluids over a moving vertical plate. The external flow and the stretching velocities are assumed to be constant. It is assumed that the plate moves in the same or opposite direction to the free stream. Using the local similarity method, it has been shown that the dual solutions of velocity and temperature fields exist for certain values of suction/injection, mixed convection, nanoparticle volume fraction and velocity ratio parameters. The non-linear ordinary differential equations along with the boundary conditions form a two point boundary value problem and are solved using Shooting method, by converting into an initial value problem. The initial value problem for a final set of first order system of ordinary differential equations is solved by fourth-order Runge–Kutta method. Three different types of nanoparticles, namely Copper (Cu), Aluminum Oxide (Al_2O_3), and Titanium Oxide (TiO_2) are considered by using water-based fluid with Prandtl number Pr = 6.2. The effect of the solid volume fraction parameter φ of nanofluids on the heat transfer characteristics is also investigated. The results indicate that dual solutions exist when the plate and the free stream moves in the same as well as in the opposite direction. The effects of various parameters on the velocity and temperature profiles are also presented.

1. Introduction

Low thermal conductivity is often the primary limitation for conventional fluids such as water, oil, ethylene glycol mixture in enhancing the performance and the compactness of many engineering electronic devices. To overcome this drawback there is a strong motivation to develop advanced heat transfer fluids with substantially higher conductivities to enhance thermal characteristics, suspension of colloidal particles dubbed as nanofluids. Choi [1] in his pioneered work uses nanoparticles which are dispersed into water and other fluids. Nanofluid, which is a mixture of nano-sized particles suspended in a base fluid, has a higher thermal conductivity than the base fluid. These suspended nanoparticles can change the transport and thermal properties of the base fluid. This higher thermal conductivity enhances rate of heat transfer in industrial applications, such as nuclear reactors, transportation, electronics as well as biomedicine and food. Nanofluids have attracted attention as a new generation of conventional fluids in building heating in heat exchangers, in plants and in automotive cooling applications, because of their excellent thermal performance. Various benefits of the application of nanofluids include improved heat transfer, heat transfer system size reduction, minimal clogging, micro channel cooling, and miniaturization of systems [1].

Nanofluids consist of very small sized solid particles and therefore in low solid concentration, it is reasonable to consider the flow of nanofluids as a single phase flow [2]. Nanofluids are also important for the production of nano structured materials for the engineering of complex fluids, as well as for cleaning oil from surfaces due to their excellent wetting and spreading behavior [3]. The comprehensive references on nanofluids can be found in the recent book [4] and in the review papers by Buongiorno [5], Wang and Mujumdar [6], Kakac and Pramuanjaroenkij [7]. Relevant experimental data [8] has shown that nanofluids possess superior heat transfer properties compared to those of ordinary fluids. Such studies also indicate that these suspensions are relatively stable because of the very small particle size. Thus, nanofluids can constitute a promising alternative for advanced thermal applications, in particular for micro/nano-heat transfer applications where the current tendency towards smaller and higher heat exchanger systems is considerable [9,10]. Choi et al. [11] showed that the addition of small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increases the thermal conductivity of the fluid up to approximately two times. The convective heat transfer characteristics of nanofluids depends on the thermophysical properties of the base fluid and the ultra fine particles, the flow pattern and flow structure, the volume

* Corresponding author. Tel.: +91 919444859677.
E-mail addresses: subha@annauniv.edu (S.V. Subhashini), sumathydhanesh@gmail.com (R. Sumathi).

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Nomenclature

a, b	constants
C_f	skin friction coefficient
C_p	specific heat at constant pressure (J K^{-1} kg^{-1})
f	dimensionless streamfunction
F	dimensionless velocity
g	acceleration due to gravity (ms^{-2})
Gr_x	local Grashof number
k	thermal conductivity
Nu_x	local Nusselt number
Pr	Prandtl number, $\mathrm{Pr} = \nu_f/\alpha_f$
Re_x	local Reynolds number based on x, $\mathrm{Re}_x = Ux/\nu_f$
T	fluid temperature in the boundary layer (K)
U	composite velocity (m s^{-1}), $U = u_w + u_e$
u, v	velocity components in the x- and y-directions, respectively (m s^{-1})
u_e	velocity of the external flow (m s^{-1})
u_w	velocity of the stretching surface (m s^{-1})
x, y	Cartesian coordinates measured along the surface and normal to it, respectively (m)

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β	volumetric coefficient of thermal expansion (K^{-1})
ε	velocity ratio parameter, $\varepsilon = u_w/U$
η	similarity variable
θ	dimensionless temperature
λ	mixed convection parameter
μ	dynamic viscosity (kg m^{-1} s^{-1})
ν	kinematic viscosity (m^2 s^{-1})
ρ	fluid density (kg m^{-3})
φ	nanoparticle volume fraction
ψ	dimensional streamfunction (m^2 s^{-1})

Subscripts

e	condition at the edge of the boundary layer
w	condition at the wall
∞	freestream condition
nf	nanofluid
f	fluid
s	solid

fraction of the suspended particles, the dimensions and the shape of these particles.

Free convection is caused by the temperature difference of the fluid at different locations and forced convection is the flow of heat due to the cause of some external applied forces. The combination of free convection and forced convection is called as mixed convection. Some examples include solar receivers exposed to wind currents, electronic devices cooled by fans, nuclear reactors cooled during emergency shutdown, heat exchanges placed in a low-velocity environment, flows in the ocean and in the atmosphere. Boundary layer flow over a stretching surface is an important problem in many engineering processes, for example aerodynamic extrusion of plastic sheets, melt spinning, the hot rolling, wire drawing, glass fiber, and paper production, manufacture of plastic and rubber sheets. In these cases, the final product of desired characteristics depends on the rate of cooling in the process and the process of stretching. Emmanuel Sanjayanand and Sujit Kumar Khan [12] have investigated on heat and mass transfer in a visco-elastic boundary layer flow over an exponentially stretching sheet. The boundary layer flow of a nanofluid caused by a stretching surface has drawn the attention of many researchers [13–20]. Mustafa et al. [21] have studied the stagnation-point flow of a nanofluid towards a stretching sheet. In recent studies, [22,23], Van Gorder et al. [22] have reported the possibility of multiphase flow in a vertical channel while applying the slip condition to provide lubrication of one boundary. Also, Loganathan et al. [24] have studied the radiation effects on unsteady natural convection flow of nanofluids past an infinite vertical plate. Ahmad et al. [25] have extended the classical forced convection boundary layer flow past a static semi-infinite flat plate (Blasius problem) and respectively, the forced convection boundary layer flow past a moving flat plate (Sakiadis problem) to the case of nanofluids. Norfifah Bachok et al. [26] have extended the Blasius and Sakiadis problems in nanofluids considered in [25], by considering a uniform free stream parallel to a fixed or moving flat plate. There are some recent studies on boundary layer flows over stationary/moving vertical plate under various thermal boundary conditions [27–29]. It may be remarked that the investigation on the existence of dual solutions in such flow situations may bring more insight on engineering applications. Further, it is noted that the upper branch solution are linearly stable while those on the lower branch are linearly unstable. In recent studies, Subhashini et al. [30,31] have reported that the upper branch solutions are most physically relevant solution whereas the lower branch solutions seem to deprive physical significance or may have realistic meaning in different situations.

The aim of the present paper is to study the thermal diffusive mixed convection flow on a moving plate in nanofluids. Three different nanofluids made of Copper (Cu), Aluminum Oxide (Al_2O_3), and Titanium Oxide (TiO_2) are tested to investigate the effect of the solid volume fraction parameter φ of nanofluids with the Prandtl number Pr = 6.2 on the flow and heat transfer characteristics. Nanofluids made using single-phase method showed higher conductivity enhancement than the ones made using two-step method. Although the two-step method works well for oxide nanoparticles, it is not as effective for metal nanoparticles such as copper. For nanofluids containing high-conductivity metals, it is clear that the single-step technique is preferable to the two-step method. Equipment with multiphase heat transfer requires special consideration, and runs the risk of overheating, leading to failure if proper care is not taken in design. In most nanofluids prepared by the two-step process, the agglomerates are not fully separated, so nanoparticles are dispersed only partially. Although nanoparticles are dispersed ultrasonically in liquid using a bath or tip sonicator with intermittent sonication time to control over heating of nanofluids, this two-step preparation process produces significantly poor dispersion quality. Because the dispersion quality is poor, the conductivity of the nanofluids is low. The locally similar solutions of the coupled non-linear partial differential equations governing the mixed convective flow have been obtained numerically using Shooting method [32] to represent dual solutions. The results for some particular cases are compared with those of Ahmad et al. [25] and Norfifah Bachok et al. [26].

2. Analysis

A steady two dimensional laminar mixed convection boundary layer flow due to a moving vertical plate in viscous and incompressible nanofluids of constant ambient temperature T_∞ (see Fig. 1) is considered. The fluid is water-based nanofluids containing different types of nanoparticles such as Copper (Cu), Aluminum Oxide (Al_2O_3), and Titanium Oxide (TiO_2). The free stream and