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Numerical solution of buoyancy MHD flow with magnetic potential

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ABSTRACT

In this study, dual reciprocity boundary element method (DRBEM) is applied for solving the unsteady flow of a viscous, incompressible, electrically conducting fluid in channels under the effect of an externally applied magnetic field and buoyancy force. Magnetohydrodynamics (MHD) equations are coupled with the energy equation due to the heat transfer by means of the Boussinessq approximation. Then, the 2D non-dimensional full MHD equations in terms of stream function, temperature, magnetic potential, current density and vorticity are solved by using DRBEM with implicit backward Euler time integration scheme. Numerical results are obtained utilizing linear boundary elements and linear radial basis functions approximation for the inhomogeneities, in a double lid-driven staggered cavity and in a channel with backward facing step. The results are given for several values of problem parameters as Reynolds number (*Re*), magnetic Reynolds number (*Rem*), Hartmann number (*Ha*) and Rayleigh number (*Ra*). With the increase in *Rem*, both magnetic potential and current density circulate near the abrupt changes of the walls. The increase in *Ha* suppresses this perturbation, and forces the magnetic potential lines to be in the direction of the applied magnetic field. The boundary layer formation through the walls emerge in the flow and current density for larger values of *Ha*.

1. Introduction

MHD deals with the interaction between the electromagnetic fields and conducting fluids. This branch of science has a wise content of it, and can be found in many references as in [9,26]. MHD flow and buoyancy-driven MHD flow have important applications in control of nuclear engineering thermo-hydraulics processes, MHD energy systems, MHD generators and magneto-plasma dynamics.

There are lots of experimental and numerical studies on incompressible fluid flow defined by Navier–Stokes equations, natural or forced convection flows, and these type of flows under an applied magnetic field (MHD flow). In most of the MHD flow studies, the induced magnetic field in the fluid is neglected due to the small magnetic Reynolds number. Applications are given mostly in cavities and some of them also include backward-facing step (BFS) flow. BFS has attracted a great deal of attention due to its important role in the design of many heat transfer devices, such as cooling systems for electronic equipment, high performance exchangers, chemical processes and energy system equipment etc. Flow in thermally stratified and isothermal lid-driven cavities has been the subject of extensive investigations at the environmental fluid mechanics. Being a complex geometry, staggered double lid-driven cavity as a combination of backward-facing step and lid-driven cavity is also remarkable.

For solving Navier–Stokes equations, DQM and wavelet-based discrete singular method (DSC) are used by Meraji et al. [21] and Zhou et al. [33], respectively, in a staggered double lid-driven cavity. Nithiarasu and Liu [25] also handled the same geometry with the explicit characteristic-based split (CBS) scheme in view of steady and unsteady flows inside a non-rectangular double lid-driven cavity. Biswas et al. [5] investigated the laminar incompressible BFS flow for a wide range of Reynolds number and aspect ratios. BFS flow at $Re = 800$ is analyzed by using a spectral domain decomposition method in [15], and it is found that the BFS flow at $Re = 800$ was stable and steady. Incompressible BFS flow equations in terms of stream function and vorticity have also been solved employing a second-order implicit, unconditionally stable finite difference method, Differential quadrature method (DQM) and multidomain boundary element method in [12,28,32], respectively.

In the numerical investigation of buoyancy-induced flow in BFS, some outstanding studies may be mentioned. Khanafer et al. [16] concentrates on the laminar mixed convection pulsating flow past a BFS using finite element method (FEM) based on Galerkin

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method of weighted residuals. Barrios-Pina et al. [4] examines the mixed convection flow over a backward-facing step passing from steady to chaotic behavior. They found that the chaotic behavior occurs when $Gr > 56600$ and $Re > 238$. With an expansion ratio of 2 in BFS, buoyancy-assisting mixed convection flow is presented by Lin et al. [19] using the finite difference scheme. The characteristics of heat transfer in the fluid region coupled with heat transfer in the solid region are studied by Kanna and Das [14] using alternating direction implicit (ADI) method. Double diffusive mixed convection flow in a BFS employing FEM by means of velocity–vorticity formulation is given by Kumar et al. [18]. Later, Kumar and Dhiman [17] also analyzed the laminar forced convection flow in an adiabatic circular cylinder inserted BFS using FLUENT. Aydin [3] has applied the stabilized subgrid FEM to the natural convection flow in BFS geometry among the different geometries. Considering the BFS as an enclosure, Chang and Tsay [8] studied the laminar natural convection flow using SIMPLER scheme.

There are some applications for fully developed MHD flow problems in channels or cavities when the equations are restricted to a plane which is perpendicular to the direction of the fluid motion. A stabilized FEM solution of the steady MHD flow problem was given by Neslitürk and Tezer-Sezgin [24] for high values of Hartmann number. Bozkaya and Tezer-Sezgin [6] have used BEM with a fundamental solution for coupled equations for solving MHD flow in infinite channels and rectangular ducts. Finite volume spectral element method is carried out for solving unsteady MHD flow through a rectangular pipe using coupled equations in terms of velocity and magnetic field in [31].

Buoyancy-driven flow under the influence of an applied magnetic field neglecting the induced magnetic field is also encountered. In [20], Lo employed the DQM in a unit square cavity as well as the cavities with aspect ratios 2 and 3. Sathiyamoorthy and Chamkha [30] presented the penalty FEM with bi-quadratic rectangular elements to solve the natural convection flow of electrically conducting liquid gallium.

The influence of the externally applied magnetic field on the fluid flow is the generation of the induced magnetic field inside the flow region when the electrical conductivity and magnetic permeability of the fluid are high. In order to simulate the 2D incompressible MHD flow, Peaceman and Rachford alternating-direction implicit (ADI) scheme is performed at low magnetic Reynolds number by Navarro et al. [23]. The full MHD problem with current density and magnetic induction equations is solved for large values of magnetic Reynolds number using DRBEM in Bozkaya and Tezer-Sezgin's study [7]. Finite element method (FEM) with some stabilization techniques is used for solving incompressible MHD equations in [2,11]. Kang and Keyes [13] compares the two different formulations which are FEM with an implicit time integration scheme in terms of stream function, and a hybrid approach using velocity and magnetic fields. FEM with a stabilization technique is also used for solving 3D MHD flows by Salah et al. [29]. In the presence of heat transfer, Abbassi and Nasrallah [1] investigated the BFS MHD flow utilizing a modified control volume FEM using standard staggered grid.

The main goal of the present study is to solve full MHD equations taking into account also the heat transfer of the fluid in two physically important geometries. The purpose is to observe the flow separation which occurs with a sudden change in channels such as staggered double lid-driven cavity and backward-facing step MHD flows. The induced magnetic field in the fluid has also been solved for various values of magnetic Reynolds number. As a numerical scheme, DRBEM has been made use of for solving this most general case of MHD flow in terms of stream function, temperature, magnetic potential, current density and vorticity for the first time to the best of authors' knowledge. DRBEM treats the non-linear and non-homogeneous terms of a partial differential equation as a series of interpolating functions (radial basis functions). All the domain integrals are converted to the boundary integrals using fundamental solution of Laplace equation and Green's identities. The space derivatives and unknown boundary values of vorticity and current density are easily calculated by using DRBEM coordinate matrix. As a boundary only method, DRBEM gives solution on the boundary and some selected interior points at considerably small computational expense due to the resulting small sized discrete systems.

2. Governing equations

The two-dimensional, laminar, unsteady flow of an incompressible, viscous and electrically conducting fluid is considered in channels with heat transfer mechanism, under the effect of an external magnetic field.

MHD flows are governed by Navier–Stokes equations of fluid dynamics and Maxwell's equations of electromagnetics through Ohm's law. Buoyancy force effect is added by means of Boussinesq approximation including energy equation coupled to MHD equations. Thus, the governing equations are the continuity, momentum, magnetic induction, and energy equations given as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$\nu\nabla^2 u=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho_0}\frac{\partial P}{\partial x}+\frac{B_y}{\rho_0\mu_m}\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right) \tag{2.2}$$

$$\nu\nabla^2 v=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho_0}\frac{\partial P}{\partial y}-\frac{B_x}{\rho_0\mu_m}\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)-g\beta(T-T_c) \tag{2.3}$$

$$\frac{1}{\sigma\mu_m}\nabla^2 B_x=\frac{\partial B_x}{\partial t}+u\frac{\partial B_x}{\partial x}+v\frac{\partial B_x}{\partial y}-B_x\frac{\partial u}{\partial x}-B_y\frac{\partial u}{\partial y} \tag{2.4}$$

$$\frac{1}{\sigma\mu_m}\nabla^2 B_y=\frac{\partial B_y}{\partial t}+u\frac{\partial B_y}{\partial x}+v\frac{\partial B_y}{\partial y}-B_x\frac{\partial v}{\partial x}-B_y\frac{\partial v}{\partial y} \tag{2.5}$$

$$\alpha\nabla^2 T=\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}, \tag{2.6}$$

where ν is the kinematic viscosity, ρ_0 is the reference density, μ_m is the magnetic permeability, g is the gravitational acceleration, β is the thermal expansion coefficient, σ is the electrical conductivity, T is the temperature, and α is the thermal diffusivity. u, v and B_x, B_y are velocity and magnetic induction components of the velocity vector $\mathbf{u}=(u,v,0)$ and the magnetic field vector $\mathbf{B}=(B_x,B_y,0)$, respectively.

The continuity equation is satisfied introducing stream function ψ as $u=\partial\psi/\partial y,\ v=-\partial\psi/\partial x$. Pressure terms are eliminated by cross differentiation of Eqs. (2.3) and (2.2), and subtraction from each other. Then, the vorticity (w) equation is obtained using $\mathbf{w}=\nabla\times\mathbf{u}=(0,0,w)=(0,0,\partial v/\partial x-\partial u/\partial y)$. Also, the magnetic potential vector $\mathbf{A}=(0,0,A)$ is defined as $\mathbf{B}=\nabla\times\mathbf{A}$ in order to satisfy the solenoidal nature $\nabla.\mathbf{B}=0$, and thus $B_x=\partial A/\partial y$, $B_y=-\partial A/\partial x$. Similarly, cross differentiation of Eqs. (2.4) and (2.5) and subtraction results in current density equation for j in which $\mathbf{j}=(0,0,j)$. The current density $\mathbf{j}$ is defined as

$$\mathbf{j}=\frac{1}{\mu_m}(\nabla\times\mathbf{B}),\quad j=\frac{1}{\mu_m}\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right) \tag{2.7}$$

Thus, the differential equations for w and j, the stream function ψ and the magnetic potential A with the help of Eqs. (2.1)–(2.5) and (2.7) take the form