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Modeling of synchronous machines with magnetic saturation

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Abstract

This paper deals with a method to derive multiple models of saturated round rotor synchronous machines, based on different selections of state-space variables. By considering the machine currents and fluxes as space vectors, possible d–q models are discussed and adequately numbered. As a result several novel models are found and presented. It is shown that the total number of d–q models for a synchronous machine, with basic dampers, is 64 and therefore much higher than known. Found models are classified into three families: current, flux and mixed models. These latter, the mixed ones, constitute the major part (52) and hence offer a large choice.

Regarding magnetic saturation, the paper also presents a method to account for whatever the choice of state-space variables. The approach consists of just elaborating the saturation model with winding currents as main variables and deriving all the other models from it, by ordinary mathematical manipulations. The paper emphasizes the ability of the proposed approach to develop any existing model without exception.

An application to prove the validity of the method and the equivalence between all developed models is reported.

Keywords: Synchronous machine; Modeling; State-space vectors; Main flux saturation

1. Introduction

When magnetic saturation in ac machines is evolved, the theory of main flux saturation in d–q axes remains the best [1–8]. Because of its simplicity, it is the most used in either motoring or generating mode for synchronous or asynchronous machines.

Although, it is considered as a global way of introducing the iron saturation, compared to other methods, today, its fidelity has no contest in predicting complex ac machine operations. This can be worth checked for example in [9–14]. Of course, it reaches its limits when local phenomena have to be investigated like in some cases of diagnosis. For that purpose, the 'abc' frame and the primary equations must be handled with the known difficulties [15–17]. For all these reasons, the theory of main flux saturation in d–q axes deserves more attention.

The present paper deals mainly with a method to develop various ac machine d–q models with main flux saturation. It may be considered as an alternative to that used in [18] with the following features:

(1) It utilizes uniquely the primitive d–p equations in conjunction with the winding currents model incorporating magnetic saturation which is relatively the most known and used in the literature [1].
(2) It can be used either for induction or synchronous machines.
(3) For a given ac machine, induction or synchronous, it can be applied to describe any possible model whatever the state-space variables, unlike in [18] where some existing models cannot be obtained with the method used.

As indicated by the title, the present work is entirely devoted to modeling saturated round rotor synchronous machines in d–q axes, regardless of the choice of the state variables. In a first part, the number of models is revised and found to be much more than known [18]. Further, by assuming the machine currents and fluxes to space vector*s*, it will be shown that the number of models for a synchronous machine with basic dampers is 64. Determination of such models is adequately detailed and deeply

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discussed. Depending on the nature of state variables, found models are arranged into three families: current (6), flux (6) and mixed models (52).

In a second part, the proposed approach for developing any possible model including magnetic saturation, is presented and applied to synchronous machines. It consists of just elaborating the saturation model with stator and rotor winding currents as main variables and deriving all the other models from it, by ordinary mathematical manipulations. To practice the method evolved, simulation was carried out with a saturated synchronous machine to prove the validity of the proposed models as well as the equivalence between them.

The study encompasses the following sections. In Section 2, the basic d–q equations of a round rotor synchronous machine and related hypothesis are provided. The number of models is immediately discussed in Section 3. After that, possible models are grouped and classified into families (Section 4). The winding currents model incorporating saturation is elaborated and followed by a description of the method for deriving the remaining models, in Section 5. Derivation of some selected models is the object of Section 6. The last Sections 7–9 constitute the application, the discussion and the conclusion.

2. Basic d–q equations and hypothesis

With the following assumptions:

(1) No magnetic hysteresis, magnetizing currents and fluxes are in phase.
(2) No skin effect, winding resistances are frequency not dependent.
(3) Space higher harmonics are neglected; flux and mmfs are sinusoidal in space.
(4) Damper windings are the same in both axes.
(5) Through each winding, the linkage flux is the sum of an appropriate leakage flux independent of saturation and a main flux subject to saturation.

The space vector electrical equations of a smooth air gap synchronous machine, in the rotor reference frame, are:

$$\bar{v}_s = R_s\bar{i}_s + \frac{d\bar{\lambda}_s}{dt} + jw\bar{\lambda}_s \quad (1)$$

$$\bar{v}_r = R_r\bar{i}_r + \frac{d\bar{\lambda}_r}{dt} \quad (2)$$

$$\bar{v}_f = R_f\bar{i}_f + \frac{d\bar{\lambda}_f}{dt} \quad (3)$$

where

$$\bar{\lambda}_s = l_s\bar{i}_s + \bar{\lambda}_m \quad (4)$$

$$\bar{\lambda}_r = l_r\bar{i}_r + \bar{\lambda}_m \quad (5)$$

$$\bar{\lambda}_f = l_f\bar{i}_f + \bar{\lambda}_m \quad (6)$$

or

$$\bar{\lambda}_s = L_s\bar{i}_s + L_m(\bar{i}_r + \bar{i}_f) \quad (4a)$$

$$\bar{\lambda}_r = L_r\bar{i}_r + L_m(\bar{i}_s + \bar{i}_f) \quad (5a)$$

$$\bar{\lambda}_f = L_f\bar{i}_f + L_m(\bar{i}_s + \bar{i}_r) \quad (6a)$$

with

$$\bar{\lambda}_m = L_m\bar{i}_m \quad (7)$$

$$\bar{i}_m = \bar{i}_s + \bar{i}_r + \bar{i}_f \quad (8)$$

and

$$L_s = l_s + L_m, \qquad L_r = l_r + L_m, \qquad L_f = l_f + L_m \quad (9)$$

Definition of symbols and subscripts are given in Appendix A. All the rotor quantities are referred to the stator. Eq. (2) is that of damper windings, consisting of a kind of short-circuited cage, hence $\bar{v}_r = 0$. Due to the absence of the q-axis excitation winding, the number of equations in the d and q axes is not the same. Thus, it is more convenient to separate the space vector equations in d–q ones.

The d-axis equations are:

$$v_{ds} = R_s i_{ds} - w\lambda_{qs} + \frac{d\lambda_{ds}}{dt} \quad (10)$$

$$0 = R_r i_{dr} + \frac{d\lambda_{dr}}{dt} \quad (11)$$

$$v_f = R_f i_f + \frac{d\lambda_f}{dt} \quad (12)$$

with

$$\lambda_{ds} = L_s i_{ds} + L_m(i_{dr} + i_f) \quad (13)$$

$$\lambda_{dr} = L_r i_{dr} + L_m(i_{ds} + i_f) \quad (14)$$

$$\lambda_f = L_f i_f + L_m(i_{ds} + i_{dr}) \quad (15)$$

and

$$i_{dm} = i_{ds} + i_{dr} + i_f \quad (16)$$

$$\lambda_{dm} = L_m i_{dm} \quad (17)$$

Equations of q-axis are:

$$v_{qs} = R_s i_{qs} + w\lambda_{ds} + \frac{d\lambda_{qs}}{dt} \quad (18)$$

$$0 = R_r i_{qr} + \frac{d\lambda_{qr}}{dt} \quad (19)$$

with

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \quad (20)$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \quad (21)$$

and

$$i_{qm} = i_{qs} + i_{qr} \quad (22)$$

$$\lambda_{qm} = L_m i_{qm} \quad (23)$$