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Density maximum effects on the onset of buoyancy-driven convection in a porous medium saturated with cold water

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ABSTRACT

When a porous medium saturated with initially stagnant cold water around the density maximum temperature is cooled from above, the nonmonotonic density profile is developed and convection may be induced in an unstable lower layer. The initial growth rate analysis suggested that the system is initially unconditionally stable and therefore, the critical onset time exists. Here we analyze the onset of buoyancy-driven convection during the time-dependent cooling using the linear stability theory and nonlinear numerical simulation. For the linear region, the growth rates are obtained from the quasi-steady state approximation and the initial value problem approach, and they support to each other. The critical time τ_c is found as a function of the dimensionless density maximum temperature $\theta_{\max}$. Based on the linear analysis, the nonlinear numerical simulation is conducted using Fourier pseudo-spectral method. From the nonlinear numerical simulation, it is found that the longer growth period is required for the smaller $\theta_{\max}$.

1. Introduction

The onset of convective instability in a horizontal porous layer has been studied extensively since Horton and Rogers' [1] and Lapwood's [2] independent work. Sun et al. [3] extended the Horton–Rogers–Lapwood problem by considering the density inversion effect in a water layer. Yen [4] and Blake et al. [5] analyzed, both theoretically and experimentally, the density inversion effect on buoyancy-driven heat transfer. Later, Holzbecher [6] studied numerically the effect of the density inversion and temperature dependent viscosity on the onset of buoyancy-driven convection in a porous medium. Numerous extensions of this kind of problem are well summarized in Nield and Bejan's [7] book. The onset of buoyancy-driven convection in water-saturated porous media plays an important role in a wide range of systems, such as seasonal freezing and melting of soil, lakes and rivers, artificial freezing of the ground as a construction technique for supporting poor soils, insulation of underground buildings, melting of the upper permafrost in the Arctic due to buried pipelines, thermal energy storage in porous media and storage of frozen foods. Most of the above applications involve developing nonlinear basic temperature profiles.

Caltagirone [8] analyzed the onset of buoyancy-driven convection in a fluid-saturated porous medium under the transient temperature. He employed the linear amplification theory and the energy method to analyze the stability characteristics. Recently, in the connection of carbon dioxide sequestration process, Ennis-King et al. [9] revisited Caltagirone's work. Riaz et al. [10] analyzed the onset of convection in porous media under the transient concentration field in self-similar coordinates. Their basic idea is quite similar to the propagation theory [11], where the penetration depth is used as a length scaling factor. Selim and Rees [12], Hassanzadeh et al. [13] and Kim and Choi [14] reconsidered the transient problem by employing the linear stability analysis, direct numerical simulation and the modified energy method, respectively. The above mentioned works have been conducted under the Boussineq approximation.

For the porous medium saturated by the cold water, Poulikakos [15] extended Caltagirone's [8] analysis by considering the density inversion effect. Poulikakos [15] employed the linear amplification theory to analyze the onset of convection which is quite popular but it involves arbitrariness in choosing both the initial conditions and the amplification factor to mark the onset of buoyancy-driven motion. Later, Kim et al. [16] extended their propagation theory to consider the density maximum effect on the onset of buoyancy-driven convection. They reformulated the stability equation in the semi-infinite domain and rescaled the disturbance quantities adopting the thermal penetration depth as a length scaling factor. Recently, many researchers [17,18] analyzed the effect of the nonmonotonic density profile on the onset of convective motion. In these mass transfer systems, the nonmonotonic profiles are due

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Nomenclature

a	dimensionless wavenumber
a^*	modified dimensionless wave number, $a\sqrt{\tau}$.
c	specific heat (J/(kg K))
$\mathbf{g}$	gravitational acceleration vector (m/s^2)
J	dimensionless heat flux
K	permeability (m^2)
k	thermal conductivity (W/(m · K))
L_c	convective length scale, $L_c = \alpha\nu/\{\varepsilon((\rho c)_f/(\rho c)_s)g2\gamma(\Delta T)^2K\}$
P	pressure (Pa)
T	temperature (K)
t	time (s)
$\mathbf{U}$	velocity vector (m/s)
w	dimensionless vertical velocity component
(x, y, z)	dimensionless Cartesian coordinates

Greek letters

α	thermal diffusivity (m^2/s)
γ	constant in Eq. (4)
ε	porosity
ζ	similarity variable, $(z/\sqrt{\tau})$
θ	dimensionless temperature, $(T_i - T)/(T_i - T_u)$
μ	viscosity (Pa s)
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)
σ	growth rate
τ	dimensionless time, $(\alpha t/d^2)$
ψ	stream function
ω	vorticity

Subscripts

c	critical condition
max	density maximum condition
0	basic quantity
1	perturbed quantity

to the double diffusion and chemical reaction. Therefore, systematic linear and nonlinear analyses on the effect of non-monotonic density profile on the onset of convective motion in the porous medium is needed to understand the related process.

In this study, time-dependent cooling of the water layer near the density-maximum temperature is analyzed using linear and nonlinear theory. When the temperature profiles in the water-saturated porous layer vary with time, the onset condition of buoyancy-driven convection is analyzed. In the present study, the time-dependent linearized disturbance equations are transformed similarly by using a similarity variable and are solved using the initial growth rate analysis and the quasi-steady state approximation. Based on the linear analysis, the nonlinear numerical simulation is also conducted employing the pseudo-spectral method. Therefore, the present study might complement the previous work for the Boussinesq fluids and give some insight into the nonmonotonic density effects on the buoyancy-driven phenomena.

2. Governing equations

The system considered here is the semi-infinite porous layer saturated with cold water of an initial temperature $T_i(>T_{\max})$, as shown in Fig. 1. Here $T_{\max}$ is the density-maximum temperature. For time $t \geqslant 0$, the horizontal porous layer is cooled from above with a constant temperature $T_u(<T_i)$. The porous layer is regarded as a homogeneous and isotropic one with the heat capacity $(\rho c)_e = \varepsilon(\rho c)_f + (1-\varepsilon)(\rho c)_s$ and thermal conductivity $k_e = \varepsilon k_f + (1-\varepsilon)k_s$, where ρ, c, k and ε represent the density, the heat capacity, the thermal conductivity and the porosity of the porous layer, respectively. The subscripts e, f and s denote the effective value, the fluid phase and the solid matrix, respectively. In the case of $T_u < T_{\max} < T_i$, a non-monotonic density profile is possible and therefore, the governing equations in the water-saturated porous layer near the density-maximum point are given by

$$\nabla \cdot \mathbf{U} = 0, \quad (1)$$

$$\frac{\mu}{K}\mathbf{U} = -\nabla P + \rho\mathbf{g}, \quad (2)$$

$$\left(\frac{\partial}{\partial t} + \varepsilon\frac{(\rho c)_f}{(\rho c)_e}\mathbf{U}\cdot\nabla\right)T = \alpha\nabla^2 T, \quad (3)$$

$$\rho = \rho_{\max}\left[1 - \gamma(T - T_{\max})^2\right], \quad (4)$$

where $\mathbf{U}$ denotes the velocity vector, μ the viscosity, K the permeability, P the pressure, T the temperature, $\mathbf{g}$ the gravitational acceleration vector, t the time, and $\alpha(=k_e/(\rho c)_e)$ the effective thermal diffusivity, $\gamma = 8\times10^{-6}(°\text{C})^{-2}$ [5] and $T_{\max} = 3.98$°C. Eq. (2) is the well-known Darcy equation including the gravity force. If the solid matrix is composed of low heat capacity materials, the value of $\varepsilon(\rho c)_f/(\rho c)_e$ can be assumed to be 1, that is, the effect of the heat capacity of the solid matrix is neglected. The objective of the present study is to find the critical condition to mark the onset of buoyancy-driven instability.

For the present stability analysis, $L_c = \alpha\nu/\{\varepsilon((\rho c)_f/(\rho c)_s)g2\gamma(\Delta T)^2K\}$ is chosen as the convective length scale, where ν denotes the kinematic viscosity. Using L_c^2/α and $\Delta T = (T_i - T_u)$ as the time and temperature scales, the basic conduction state is represented in dimensionless form by

$$\frac{\partial\theta_0}{\partial\tau} = \frac{\partial^2\theta_0}{\partial z^2}, \quad (5)$$

under the following boundary conditions:

$$\theta_0 = 0 \quad \text{at} \quad \tau = 0 \quad \text{and} \quad z \to \infty \quad (6a)$$

$$\theta_0 = -1 \quad \text{at} \quad z = 0. \quad (6b)$$

Here, $\theta_0 = (T - T_i)/\Delta T$. The solution of the above equation is given in Carslaw and Jaeger's [19] book;

$$\theta_0 = -erfc\left(\frac{\zeta}{2}\right), \quad (7)$$

where $\zeta = z/\sqrt{\tau}$. Based on this temperature distribution, the following dimensionless density distributions are given in Fig. 2:

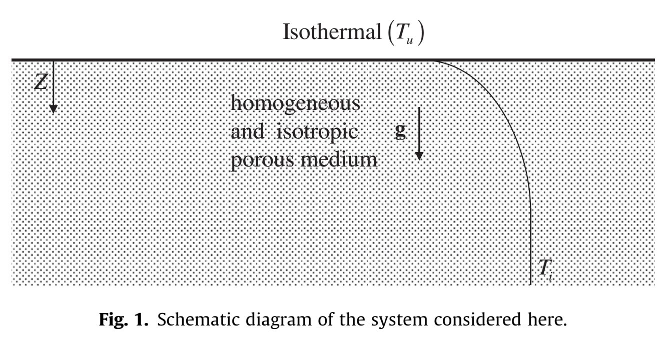

Fig. 1. Schematic diagram of the system considered here.