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High accuracy numerical investigation of double-diffusive convection in a rectangular enclosure with horizontal temperature and concentration gradients

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ABSTRACT

Double-diffusive convection of a binary mixed fluid in a rectangular enclosure with horizontal temperature and concentration gradients is numerically investigated. The problem with thermal Rayleigh numbers of 10^4 and 10^5, the Prandtl number being range of [0.015, 12], the Lewis number being range of [0.05, 100], the buoyancy ratio of 0.8, and the height-to-width aspect ratio of 2 are considered. A compact difference method with fully fourth-order accuracy and high resolution in space, involving a, at least, fourth-order upwind compact scheme suggested for approximation of the nonlinear convective terms, a fourth-order symmetrical compact scheme to discretize the viscous terms and the third-order TVD Runge–Kutta method employed for time discretization, is proposed for solving this unsteady double-diffusive convection problem. The effects of Prandtl and Lewis numbers on flow structure, the temperature and concentration distribution are investigated and discussed for $Ra = 10^4$ and 10^5 respectively in a rectangular enclosure with the aspect ratio $A = 2$. A bifurcation for the double-diffusive convection is captured with the variation of Prandtl number, and two different types of periodic flow motion with Lewis number in different ranges are observed.

1. Introduction

Double-diffusive convection occurs in a fluid when the density variations is caused by two different components with different rates of diffusion, viz. natural convection driven by buoyancy due to simultaneous temperature and concentration gradients. This type of natural convection exists not only in many natural phenomena but also in many procedures of industrial applications, for example, atmosphere, ocean circulation, asthenosphere movement within crust, pollution transportation in air, water, soil system, physical and chemical vapor transportation [7,14,16,17] and convection in crystal-growth processes for semi-conductors and in solidification of metallic alloys and so on [3,5,6,20]. For a detailed survey of literature on the double-diffusive convection, see Trevisan and Bejan [8], Mamou et al. [10] and Ghorayeb et al. [27].

The double-diffusive convection is a strong nonlinear coupled problem, which has been investigated by many researchers [1,8–12,15,17,27,38]. Stommel [1] obtained multiple steady-state solution for double-diffusive convection by doing researches of the concentration and thermal effect on density field, and also proved the existence of multiple equilibrium states is due to mixed boundary condition. Stern [2] qualitatively put forward that the uneven concentration and thermal layers of fluid field will cause internal separation and formation of vortices. Quon and Ghil [18] obtain numerical results of multiple steady-state with mixed boundary condition during the research of long periodic motion of the Atlantic and global ocean currents. Thualhe and Mcwilliams [19] indicated that multiple steady-state could be numerically obtained by increasing default salt flux. Recently, with the development of semi-conductors and alloys, there has been increasing interests in flow structure of double-diffusive convection in a cavity. Many numerical researches using the different numerical methods have been carried out. Nishimura et al. [26] explored the effect of buoyancy ratio in double-diffusive convection in a rectangular enclosure with combined horizontal temperature and concentration gradients by using Galerkin finite element method, found that oscillatory flow occurs in a limited range of buoyancy ratio, and claimed the themosolutal instability is the mechanism of oscillatory flow. By using a finite difference method with second-order accuracy, Ghorayeb et al. [27] exhibited the effect of Lewis number in range of 2–45 on double-diffusive convection in a square cavity with combined horizontal temperature and concentration gradients, and indicated that the value of Lewis number decides the oscillatory flow will be asymmetric or centro-symmetric. Fluid,

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heat and contaminant transport structures of laminar double diffusive mixed convection in a 2D ventilated enclosure were numerically investigated by Deng et al. [11]. The use of a linear stability analysis, Malashetty et al. [12] studied analytically and numerically the effect of thermal modulation on the onset of double-diffusion natural convection in a horizontal fluid layer and found that high frequency symmetric modulation was always stabilizing while the frequency symmetric modulation was destabilizing. However, the works mentioned above did not discuss much of the details of fluid field.

As a strong nonlinear coupled problem, the process of the double-diffusive convection includes multi-scale structures and time dependent behaviors. Even the numerical simulation of double-diffusive convection in a simple rectangular enclosure also requires a great deal of computer efforts, therefore it is important to design efficient numerical scheme with high accuracy and resolution. High-order compact (HOC) difference methods feature high accuracy and narrower stencils and combine reasonable computational costs and flexibility in the use of boundary conditions, which are nowadays very popular in the fluid dynamics community, see for examples [22,25,28,29,31,34,35]. Compact finite difference schemes in general have better resolving capabilities for small wavelength phenomena than standard finite difference schemes of the same order and their capability to solve various complicated problems have been proven, see e.g., [22–24,28,34,38]. To the best of knowledge of the authors, however, the implementation of the HOC scheme to perform the double-diffusive convection problems is very few [38]. In [38], the convective terms (viz. the first derivatives) and the diffusive terms (viz. the second derivatives) in the vorticity, the temperature and the concentration equations are discretized using the 7-points *seven-order* upwind compact schemes proposed and the 5-points sixth-order symmetrical compact schemes, respectively. Furthermore, the 5-points *fifth-order* upwind and the 3-points *third-order* upwind scheme are employed for approximating convection terms adjacent to the boundary. Strictly speaking, the HOC method in [38] is *only third-order accuracy* due to use the third-order upwind scheme for approximating convection term adjacent to the boundary. In this paper, based on the properties of numerical dispersion relation preservation (DRP) [40], a compact difference method with *full fourth-order accuracy* and high resolution in space is firstly suggested for simulating the double-diffusive convection in a rectangular enclosure with horizontal temperature and concentration gradients. Then, the numerical capability and code of the developed method is confirmed by the application to natural convection in a square enclosure for different Rayleigh numbers Ra (from 10^4 to 10^6) and double-diffusive convection in a rectangular enclosure with the aspect ratio $A = 2$ for Rayleigh number $Ra = 10^5$, Prandtl number $Pr = 1$ and the Lewis number $Le = 2$. Compared with some of the accurate results available in the literature, the obtained numerical results demonstrate the validation of accuracy and effectiveness of the currently proposed method. Furthermore, the details of different flow structures are discussed for wide range of Prandtl numbers, $0.015 \leqslant Pr \leqslant 12$ and Lewis numbers, $0.05 \leqslant Le \leqslant 100$ for Rayleigh numbers $Ra = 10^4$ and 10^5 in a rectangular enclosure with the height-to-width aspect ratio $A = 2$.

This article is organized as follows. In the next section, the physical modeling, governing equations, boundary conditions and dimensionless parameters are introduced. In Section 3, a new high-order finite difference discretization method is suggested for the double-diffusive convection problem, involving a newly proposed at least fourth order accurate upwind compact scheme with high resolution for approximation of nonlinear convective terms. In Section 4, the results of the numerical simulations are reported, including the effects of Prandtl numbers and Lewis numbers on flow structure and temperature and concentration distribution and finally, conclusions are summarized in Section 5.

2. Model description and governing equations

As shown in Fig. 1, the problem being solved is the convection of a binary mixed fluid in a rectangular enclosure, whose height and width are given as H and W respectively. In the present study, both horizontal walls are considered to be adiabatic as well as impermeable, while the constant temperatures and concentrations are maintained on the vertical walls. The temperature of left wall T_h is hotter than that of right wall T_l, while the concentration of left wall C_h is higher than that of right wall C_l. To study this problem, we take following assumptions: the fluid is Newtonian in behavior; the viscous dissipation is negligible; thermal properties are independent; Boussinesq approximation is taken to describe the density of fluid. The Boussinesq approximation assumes the density of fluid ρ only effected by temperature T and concentration C, so the function of ρ is given as

$$\rho = \rho_0(1 - \beta_T(T - T_0) + \beta_C(C - C_0)) \tag{1}$$

where β_T and β_C are the thermal expansion and the concentration expansion coefficients respectively, T_0, C_0 and ρ_0 are the reference temperature, the concentration and the density, respectively. The two dimensional (2D) governing equations to describe this problem are given as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\rho}{\rho_0}g \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa_T\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \kappa_C\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{6}$$

where u and v are the velocity components in x and y direction respectively, p is pressure, g is the gravity acceleration. The kinematic viscosity υ, the thermal diffusivity κ_T, the concentration diffusivity κ_C are determined by the properties of this binary mixed

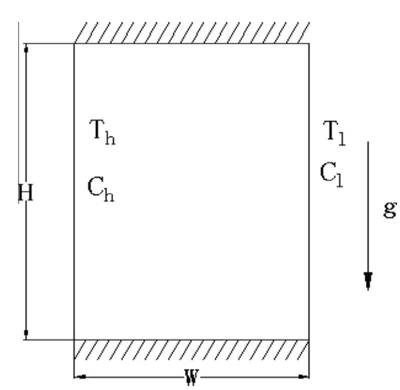

Fig. 1. Schematic diagram of the problem with the boundary conditions.