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Non-similar solution for natural convective boundary layer flow of a nanofluid past a vertical plate embedded in a doubly stratified porous medium

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ABSTRACT

This article studies the effects of thermal and mass stratification on natural convection boundary layer flow over a vertical plate embedded in a porous medium saturated by a nanofluid. The plate is maintained at a uniform and constant wall temperature, concentration and nanoparticle volume fraction. The effects of Brownian motion and thermophoresis are incorporated into the model for nanofluids. In addition, the thermal energy equations include regular diffusion and cross-diffusion terms. A suitable coordinate transformation is introduced, and the obtained system of non-similar, coupled and nonlinear partial differential equations are solved by the Keller-box method. The effect of the Brownian motion parameter, thermophoresis parameter, modified Dufour number, Dufour-solutal Lewis number, thermal and mass stratification parameters on the non-dimensional velocity, temperature, concentration, nanoparticle volume fraction, heat and mass transfer rates at the plate are discussed and displayed graphically.

1. Introduction

The study of nanofluids has gained much interest due to its exceptional applications in electronics, communication, computing technologies, optical devices, lasers, high-power X-rays, scientific measurement, material processing, medicine and material synthesis. The term nanofluid was first used by Choi [1] to refer to a fluid with suspended nanoparticles. Nanofluids are prepared by dispersing solid nanoparticles in fluids such as water, oil, or ethylene glycol. Choi et al. [2] showed that the addition of a small amount (less than 1 by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid. The random motion of nanoparticles within the base fluid is called Brownian motion, and this results from continuous collisions between the nanoparticles and the molecules of the base fluid. Particles can diffuse under the effect of a temperature gradient. This phenomenon is called thermophoresis, and is the "particle" equivalent of the well-known Soret effect for gaseous or liquid mixtures. The detailed introduction and applications of nanofluids can be found in the book by Das et al. [3]. Buongiorno [4] has investigated the factors which contribute to abnormal thermal conductivity increase relative to base fluids and viscosity. He developed an analytical model for convective transport in nanofluids, which takes Brownian diffusion and thermophoresis effects into account. The literature on nanofluids has been reviewed by Daungthongsuk and Wongwises [5], Wang and Mujumdar [6], Kakac and Pramuanjaroenkij [7], Gianluca et al. [8] among several others. These reviews discuss in detail the work done on convective transport in nanofluids.

Convective transport in porous media has been the subject of great importance and interest in recent years owing to its wide range of applications in civil, chemical and mechanical engineering. Several authors investigated the mixed convection heat and mass transfer along non-isothermal vertical surface embedded in a nanofluid saturated porous medium. Kuznetsov and Nield [9] studied the natural convective boundary-layer flow of a nanofluid past a vertical plate analytically. Khan and Pop [10] investigated numerically the problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid that incorporates the effects of Brownian motion and thermophoresis. Kuznetsov and Nield [11] analytically studied the double-diffusive natural convective boundary-layer flow of a nanofluid past a vertical plate. Hamad [12] examined the convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of a magnetic field. Chamkha et al. [13] presented boundary layer analysis for the non-similar solution of natural convection past an isothermal sphere in a Darcy porous medium saturated with a nanofluid. Aziz and Khan [14]

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Nomenclature

B	regular buoyancy parameter
$C_\infty(x)$	ambient stratified concentration
C	concentration
Da	Darcy parameter ($=K_p/L^2$)
D_B	Brownian motion diffusion coefficient
D_{CT}	Soret type diffusivity
D_s	mass diffusivity
D_T	thermophoretic diffusion coefficient
D_{TC}	Dufour type diffusivity
f	dimensionless stream function
g^*	acceleration due to gravity
g	dimensionless Nanoparticle fraction
Gr	Grashof number
k	thermal conductivity
K_p	permeability
L_d	Dufour-solutal Lewis number
N_b	Brownian motion Parameter
N_d	modified Dufour number
N_r	nanofluid buoyancy parameter
N_t	thermophoresis parameter
Nu_ξ	non-dimensional Nusselt number
Pr	Prandtl number ($=\nu/\alpha$)
s	dimensionless concentration
Sc	Schmidt number ($=\nu/D_s$)
Sc_n	nanoparticle Schmidt number ($=\nu/D_B$)
Sh_ξ	non-dimensional Sherwood number
$T_\infty(x)$	ambient stratified temperature
T	temperature
u, v	velocity components in the x- and y-directions respectively.
x, y	cartesian coordinates along the plate and normal to it

Greek symbols

β_T	coefficient of thermal expansion
β_c	coefficient of concentration expansion
ρ_p	density of nanoparticles
ρ_f	density of the base fluid
θ	dimensionless temperature
ν	kinematic viscosity
ϕ	nanoparticle volume fraction
ξ	non-similarity variable
η	similarity variable
ε_2	solutal stratification parameter
ψ	stream function
α	thermal diffusivity
ε_1	thermal stratification parameter
μ	viscosity of the fluid

Subscripts

w	condition at wall
∞	condition at infinity

Superscript

$'$	differentiation with respect to η

investigated the natural convective flow of a nanofluid over a convectively heated vertical plate using a similarity analysis of the transport equations followed by their numerical computations. Cheng [15] studied the natural convection boundary layer flow over a truncated cone embedded in a porous medium saturated by a nanofluid with constant wall temperature and constant wall nanoparticle volume fraction. Bachok et al. [16] performed an analysis to study the heat transfer characteristics of steady two-dimensional boundary layer flow past a moving permeable flat plate in a nanofluid. Mahdy and Ahmed [17] numerically examined laminar free convection of a nanofluid along a vertical wavy surface saturated porous medium. Kameswaran et al. [18] investigated the effects of homogeneous heterogeneous reactions in nanofluid flow over a stretching or shrinking sheet placed in a porous medium saturated with a nanofluid.

The analysis of free convection in a doubly stratified medium (stratification of medium with respect to thermal and concentration fields) is a fundamentally interesting and important problem because of its broad range of engineering applications. These applications include heat rejection into the environment such as lakes, rivers and seas, thermal energy storage systems such as solar ponds, and heat transfer from thermal sources such as the condensers of power plants. Rosmila et al. [19] examined the magnetohydrodynamic convection flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of thermal stratification. They concluded that the flow field, temperature, and nanoparticle volume fraction profiles are significantly influenced by thermal stratification and magnetic field. Anbuchezhian [20] investigated numerically the problem of laminar fluid flow, which results from the stretching of a vertical surface with variable stream conditions in a nanofluid due to solar energy.

Early studies on convection transport focussed on seeking similarity solution because similarity variables can give great physical insight with minimal effort. However, the non-similarity boundary layer flows are more general in nature in real life, and thus are more important than the similarity ones. It may be remarked that earlier studies did not include the effect of double stratification on double diffusive natural convection flow of a nanofluid past a vertical plate in a porous medium. Hence, the aim of the present investigation is to obtain non-similar solution and to consider the effect of double stratification on natural convection heat and mass transfer over a vertical plate in a nanofluid saturated porous medium.

2. Mathematical formulation

Consider the natural convection boundary layer flow over a vertical plate embedded in a porous medium saturated with a nanofluid. The x coordinate is taken along the plate, in the ascending direction and the y coordinate is measured normal to the plate, while the origin of the reference system is considered at the leading edge of the vertical plate. The plate is maintained at a uniform and constant wall temperature and concentration T_w and C_w respectively. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0} + Ax, C_\infty(x) = C_{\infty,0} + Bx$, where A and B are constants which are varied to alter the intensity of stratification in the medium. The values of T_w and C_w are assumed to be greater than the ambient temperature $T_{\infty,0}$ and concentration $C_{\infty,0}$ at any arbitrary reference point in the medium (inside the boundary layer). The nanoparticle volume fraction on the surface of the plate is ϕ_w and the ambient value of the nanoparticle volume fraction is