International Journal of Heat and Mass Transfer 71 (2014) 706–719

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Effects of heat and mass transfer on peristaltic flow in a non-uniform rectangular duct

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ARTICLE INFO

Article history:
Received 22 August 2013
Received in revised form 6 December 2013
Accepted 13 December 2013
Available online 25 January 2014

Keywords:
Peristaltic flow
Heat and mass transfer
Rectangular duct
Analytical solution

ABSTRACT

The model of bioheat transfer in tissues has attracted many researchers due to its application in thermotherapy and human thermoregulation system. Currently bioheat is considered as a heat transfer in human body. In view of this the influence of heat and mass transfer on peristaltic flow in a non-uniform rectangular duct is studied under the consideration of long wavelength $(0 \ll \lambda \to \infty)$ and low Reynolds number $(Re \to 0)$. The flow is examined in wave frame of reference moving with the velocity c. Mathematical modeling is based upon the laws of mass, linear momentum, energy and concentration. Analysis is also presented for Prandtl number Pr, Eckert number E, Schmidt number Sc and Soret number Sr. The influence of various emerging parameters of interest is seen for both two and three dimensional graphs. Numerical integration is used to analyze the novel features of volumetric flow rate, average volume flow rate, instantaneous flux and pressure gradient. The trapping bolus phenomena is also presented through stream lines.

1. Introduction

It is now well established fact that most of the physiological fluids are non-Newtonian in character [1–10]. Several models have been proposed to explain such physiological fluids in order to find the treatment of diagnostic problems that arise during the circulation in a human body. Practical applications and academic curiosity in physiology have generated a lot of interest in studying the peristaltic motion in ducts. Peristaltic mechanism deals with the fluid transport that occurs by a progressive wave of area of contraction or expansion along a length of tubes/channels. Peristaltic transport is found in living body such as movement of ovum in female fallopian tube, swallowing food through esophagus, transport of lymph in lymphatic vessels vasomation of small blood vessels like arterioles, venules, capillaries, transport of spermatozoa in ducts efferentes of male reproductive tract etc. After the experimental work of Latham [11] on peristaltic transport, several theoretical studies [12–20] have been undertaken by many researchers on peristaltic motion under one or more simplified assumptions of small amplitude ratio, small wave number, low Reynolds number and long wavelength etc.

Peristaltic flow with heat and mass transfer has many applications in biomedical sciences and industry such as conduction in tissues, heat convection due to blood flow from the pores of tissues and radiation between environment and its surface, food processing and vasodilation. The processes of oxygenation and hemodialysis have also been visualized by considering peristaltic flows with heat transfer. Obviously there is a certain role of mass transfer in all these processes. Mass transfer is important phenomenon in diffusion process such as nutrients diffuse out from the blood to neighboring tissues. Mass transfer also occurs in many industrial processes like membrane separation process, reverse osmosis, distillation process, combustion process and diffusion of chemical impurities. When the effects of heat and mass transfer are considered simultaneously then the complicated relationships occur between driving potentials and fluxes. The energy flux is induced by temperature gradient. The composition gradients and mass flux can be produced by temperature gradient which is known as Soret effect.

Investigations of heat and mass transfer in peristalsis yet have been considered by few researchers. For instance, The influence of heat transfer and magnetic field on peristaltic transport of Newtonian fluid in a vertical annulus has been discussed by Mekheimer and elmaboud [21]. Ellahi et al. [22] investigated the series solutions for magnetohydrodynamic flow of non-Newtonian nanofluid and heat transfer in coaxial porous cylinder with slip conditions. Influence of heat and mass transfer on peristaltic flow of Eyring–Powell and Jeffrey's fluids are discussed by Akbar and Nadeem [23,24]. Radhakrishnamacharya and Murty [25] studied the heat transfer on the peristaltic transport in a non-uniform channel.

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.12.038

Vajravelu et al. [26] reported the peristaltic flow and heat transfer in a vertical annulus under long wavelength approximation. A careful review of the literature reveals that a very little efforts are yet devoted to examine the peristaltic flow in a rectangular duct. Some relevant studies on the topic can be found from the list of references [27–29].

To the best of our knowledge, no attempt is made to investigate the peristaltic flow in a non-uniform rectangular duct with heat and mass transfer so far. Such consideration is very important since the heat transfer in human tissues involves complicated processes like heat transfer due to perfusion of arterial-venous blood through the pores of tissue, heat conduction in tissues, external interactions and metabolic heat generation such as electromagnetic radiation emitted from cell phones etc. Moreover, it is well known that heat and mass transfer problem in the presence of chemical reaction is very significant in the processes of geothermal reservoirs, drying, enhanced oil recovery, flow in a desert cooler, cooling of nuclear reactors thermal insulation and evaporation at the surface of a water body. These types of flows involve many practical operations such as molecular diffusion of species in presence of chemical reaction within or at boundary. Heat and mass transfer effects are also encountered in chemical industry like in the study of hot salty springs in sea, in thermal recovery processes and in reservoirs. The results obtained for title problem reveal many interesting behaviors that warrant further study on heat and mass transfer problems with chemical reaction.

The flow modeling is based on continuity, momentum and energy equations. These equations are first expressed in terms of stream function and then solved in closed form when the long wavelength and small Reynolds number assumption hold. The effects of magnetic field are taken into account. The obtained expressions are utilized to discuss the role of emerging parameters on the flow quantities. Numerical computations has been used to evaluate the expression for pressure rise. Stream lines of various interesting parameters. Finally, the effect of various emerging parameters are discussed through graphs and trapping phenomenon. This paper is arranged as follows. Section two presents the mathematical formulation for the problem of interest. Section three deals with the solution of the problem. Finally section four synthesis detailed computational results and discussion with the physical interpretation of our findings.

2. Mathematical formulation

We consider the peristaltic flow of an incompressible viscus fluid in a non-uniform duct of rectangular cross section having channel width $2d$ and height $2a + 2kX$. We choose Cartesian coordinate system in such a way that X-axis is taken along the axial direction, Y-axis is taken along the lateral direction and Z-axis is along the vertical direction of channel. The flow geometry of the problem [30] presented as Fig. 1.

Sinusoidal waves of long wavelength are assumed to travel with the velocity c along the walls of channel. These waves are

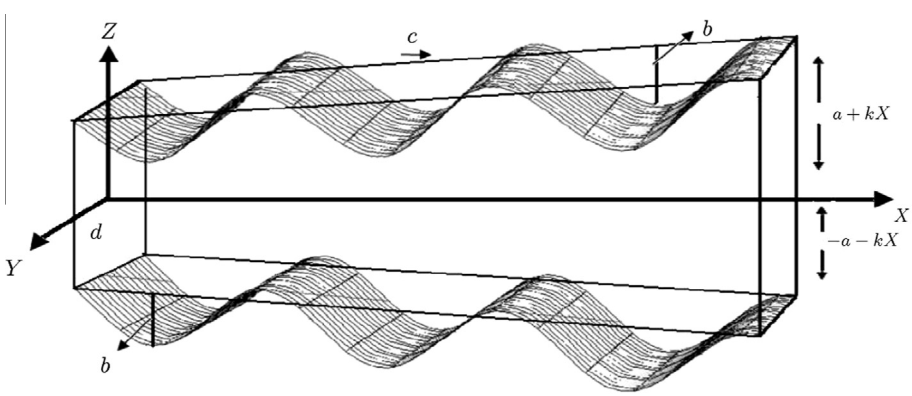

Fig. 1. Geometry of the problem.

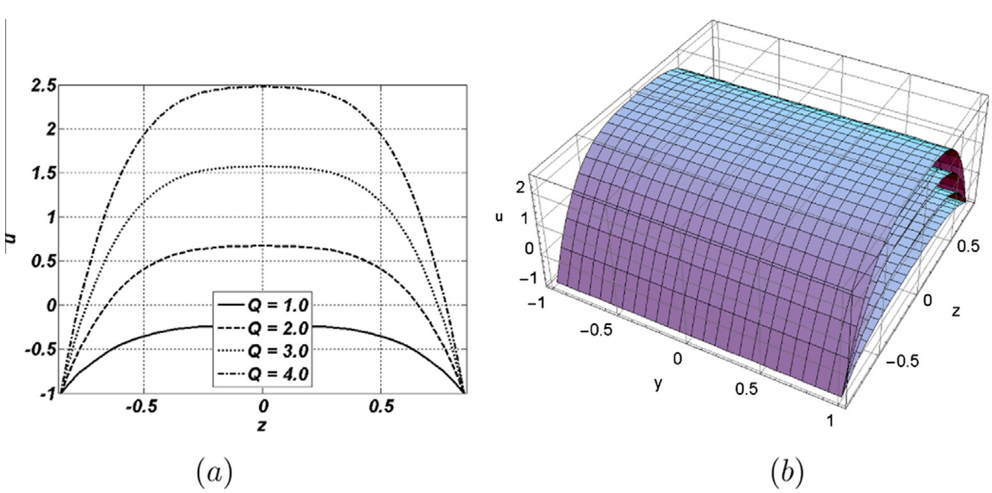

Fig. 2. Velocity profile for different values of Q for fixed $K^* = 0.9$, $\beta = 1.2$, $\phi = 0.6$ (a) for 2-dimensional (b) for 3-dimensional.