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Large eddy simulation of turbulent flow and heat transfer in a square duct with unstable natural convection on the cross section

Liang-Dong Ma [a,b], Zeng-Yao Li [a,*], Wen-Quan Tao [a]

[a] The Key Laboratory of Thermo-Fluid Science and Technology of MOE, School of Energy and Power Engineering, Xi' an Jiaotong University, Xi' an, China
[b] Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian, China

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ABSTRACT

In this paper, large eddy simulations with the dynamic Smagorinsky eddy viscosity model are performed for a fully developed turbulent flow and heat transfer in a square duct with unstable natural convection on the cross section at a turbulent Reynolds number of 400, a Prandtl number of 0.7 and different Grashof numbers from 10^5 to 5×10^7. The MPI parallel programming language is implemented to speed up the calculations. The influences of the buoyancy force on the mean flow and heat transfer, turbulent intensity and Reynolds stresses are analyzed. The results show that the mean velocity decreases while the subgrid viscosity, turbulent intensity and heat transfer increase obviously with the increase of Grashof number. The turbulent intensity in the streamwise direction decreases considerably in the near-wall region while the turbulent intensity in the spanwise direction increases clearly in almost whole regions with an increase in Grashof number. The spatial distribution of Reynolds stresses are mainly influenced by the thermal buoyancy force. The turbulence production rate and the buoyancy force production term of Reynolds stress component $\langle v'v' \rangle$ increase with the increase of Grashof number.

1. Introduction

During the last several decades, the characteristics of the turbulent flow and heat transfer in a noncircular ducts have been frequently studied by experimental and numerical methods [1–7]. In noncircular ducts, the turbulent flow and heat transfer are considerably complex owing to the second-kind secondary flow induced by the turbulent motion. Although the secondary flow is quite weak (usually only about 2–3% of the primary flow velocity), it has a great influence on the wall stress distribution and heat transfer rate [8].

For the forced turbulent flow and heat transfer in a square duct, Brundrett and Burroughs [9] performed an experimental study on the distributions of the mean temperature and revealed that the secondary flow greatly influences the local wall heat flux distribution. Hirota et al. [10] also experimentally investigated the temperature fluctuation intensity, cross-correlation coefficients between fluctuating velocity and temperature, and the turbulent heat fluxes on the cross section. The authors also discussed the characteristics of eddy-diffusivity and turbulent Prandtl number. In order to predict the turbulent-driven secondary flow, the algebraic stress model [1], the non-linear forms of k–ε equations [11] or anisotropic low-Reynolds-number k–ε turbulence model [6] have been generally used. Such models can obviously capture the secondary flow. However, the turbulent quantities are over or under predicted compared to experimental data because the secondary motion is modeled empirically.

Direct numerical simulation (DNS) is an effective tool to predict the fluctuation characteristics of turbulent flow and heat transfer. Gavrilakis [12], Huser and Biringen [13], Zhu et al. [14], Piller and Nobile [15] performed a DNS to investigate the fully developed turbulent flow in a straight square duct. These DNS results show that the secondary flow does not dramatically affect the mean friction factor and Nusselt number but the distributions of local heat flux and shear-stress at the walls, and the eddy-diffusivity approach is capable of reproducing turbulent heat flux. Yang et al. [16] performed a coarse-grid direct numerical simulation (c-DNS) to investigate the forced turbulent heat convection in a straight square duct at a bulk Reynolds number of 10^4, in which temperature was treated as a passive scalar and the buoyancy force was neglected. Ma et al. [17] used a DNS for the square duct flow with stable natural convection on a cross section. The results show that the mean flow and temperature on the cross section are strongly affected by the Reynolds stresses. The intensity of the combined heat transfer is somewhat decreased compared with that of the laminar heat transfer at the same Grashof number. Iida et al. [18,19] performed a DNS for the turbulent channel flow under stable and unstable density stratification. They reported that the internal gravity waves are built up in the core region under the stable strat-

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* Corresponding author. Tel.: +86 29 8266 5446; fax: +86 29 8266 5445.
E-mail address: lizengy@mail.xjtu.edu.cn (Z.-Y. Li).

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Nomenclature

C_s	turbulent model coefficient
c_p	specific heat at constant pressure
E_k	mean turbulent kinetic energy
Gr	Grashof number, $g\beta(T_h - T_c)H^3/\nu^2$
g	gravitational acceleration
H	width of duct
k	turbulent kinetic energy
Nu	Nusselt number, $Nu = q_w H/(\lambda \Delta T)$
N_y	number of grid points in the y-direction
Pr	molecular Prandtl number
$\mathrm{Pr_t}$	turbulent Prandtl number
q_w	the heat flux at the heated wall
q_j	subgird turbulent heat flux
$\mathrm{Re_m}$	bulk mean Reynolds number, $\mathrm{Re}_m = u_m H/\nu$
Re_τ	Reynolds number, $\mathrm{Re}_\tau = u_\tau H/\nu$
T_c	the temperature on the cold wall
T_h	the temperature on the heated wall
u, v, w	the velocity components in x-, y- and z-directions
$\left\langle \overline{u}'_i \overline{u}'_j \right\rangle$	mean Reynolds stresses
$\langle \overline{u}'_i \overline{\theta}' \rangle$	mean turbulent thermal flux
u_τ	wall friction velocity, $\sqrt{\tau_w/\rho}$
p	fluctuating pressure, $p = P/(\rho u_\tau^2)$
x, y, z	Cartesian coordinates

Greek

β	volumetric expansion coefficient
ΔT	temperature difference, $T_h - T_c$
$\overline{\Delta}$	filter width
Θ_τ	friction temperature
θ	dimensionless temperature
λ	thermal conductivity
ν	kinematic viscosity
τ_{ij}	subgrid turbulent stess
τ_w	wall shear stress
< >	ensemble average in the x-direction and in time
—	filtered variables

Subscripts

b	time and volume averaged
rms	root-mean-square fluctuation
SGS	subgrid stress
w	wall
′	fluctuating value

Superscripts

+	dimensionless variable
^	intermediate value

ification flow and the turbulent heat flux is suppressed drastically, while the unstable buoyancy affects both the statistical and instantaneous near-wall turbulent structures for the unstable stratification.

LES has been a well recognized technique. Kajishima and Miyake [20], Madabhushi and Vanka [21], Su and Friedrich [22] and Zhu et al. [23] perform a LES to study the turbulent forced flow in a straight square duct. These works correctly predict the existence of secondary flows and their effects on the mean flow and turbulence statistics. Martin et al. [24] used forced generalized Lattice Boltzmann equations (GLBE) to simulate the turbulent flow and secondary motions in a square duct. The results show that the GLBE is a reliable and accurate computational method for the complex bounded turbulent flows. For complex turbulent flow and heat transfer, Pallare and Dividson [25–27] carried out a LES on the turbulent flow and heat transfer in a rotating square duct. Tyagi and Acharya [28], Abdel-Wahab and Tafti [29], Murata and Mochizauki [30] performed a LES to investigate the fully developed turbulent flow and heat transfer in a ribbed square duct. These studies examined the effect of ribs on the flow and heat transfer. Garg et al. [31], Armenio and Sarkar [32] performed an LES investigation on the stable stratified turbulent channel flow. Recently, Dong [33] investigated the influence of temperature oscillation on characteristics of thermally stratified turbulent flow and the validity of LES technique on predicting the unsteady turbulent flow and heat transfer. These works indicate that the Smagorinsky eddy viscosity model or its dynamical form can be applied to simulate the turbulent flow and heat transfer in the complex bounded flow.

To the authors' knowledge, there has been no experimental and computational data on the turbulent flow and heat transfer in the square duct with unstable nature convection on the cross section. The objective of this work is to perform a LES with the dynamic eddy viscosity model on a fully developed turbulent flow and heat transfer in a square duct with unstable natural convection on the cross section with the unstable natural convection and investigate the effect of buoyancy force on the mean flow and temperature distribution and the turbulent characteristics. The turbulent Reynolds number (Re_τ) based on the friction velocity and the hydraulic diameter is 400, the Grashof number (Gr) is from 10^5 to 5×10^7 and the Prandtl number (Pr) is 0.71. A mixed numerical scheme is adopted to conveniently perform the parallel program. The contents include: (1) the physical and mathematical models in Section 2; (2) the mixed numerical scheme in Section 3; (3) the validation of code in Section 4; (4) the mean resolved velocities and temperature distribution, the Reynolds stresses distribution, the mean momentum and energy equation budgets in Section 5.

2. Physical and mathematical model

Fig. 1 shows the physical model and the coordinate system of the straight square duct. The flow is driven by an externally imposed mean pressure gradient in the horizontal direction and assumed to be fully developed and incompressible. The horizontal walls of square duct keep an imposed temperature difference and the vertical walls are insulated. The resolved scales and the corresponding governing equations are defined by the filtering operation based on a top-hat filter. The dimensionless filtered continuity, Navier–Stokes and energy equations under the Boussinesq approximation can be expressed as

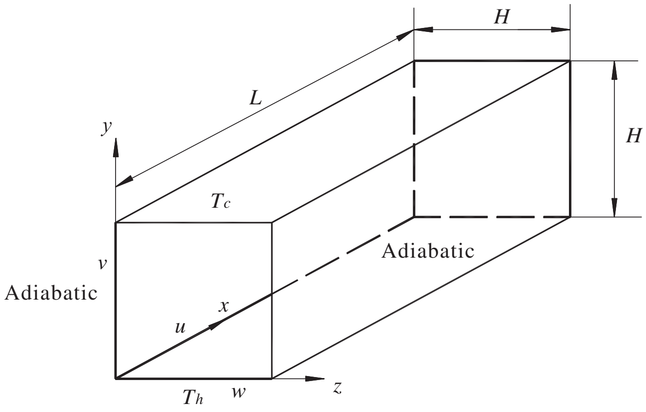

Fig. 1. Flow geometry and coordinate system.