International Journal of Heat and Mass Transfer 66 (2013) 200–209

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Laminar mixed convection flow and heat transfer characteristics in a lid driven cavity with a circular cylinder

Khalil Khanafer [a,b,*], S.M. Aithal [c]

[a] Frankel Vascular Mechanics Laboratory, Biomedical Engineering Department, University of Michigan, Ann Arbor, MI 48109, USA
[b] Section of Vascular Surgery, Samuel and Jean Frankel Cardiovascular Center, University of Michigan, Ann Arbor, MI 48109, USA
[c] Computing, Environment and Life Sciences Directorate, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439, USA

ARTICLE INFO

Article history:
Received 30 March 2013
Received in revised form 1 July 2013
Accepted 6 July 2013

Keywords:
Lid-driven cavity
Mixed convection
Circular cylinder

ABSTRACT

Mixed convection flow and heat transfer characteristics in a lid-driven cavity with a circular body inside are studied numerically using a finite element formulation based on the Galerkin method of weighted residuals. Comparisons of streamlines, isotherms and average Nusselt number are presented to show the impact of the Richardson number, non-dimensional radius of the cylinder, and the location of the cylinder on the transport phenomena within the cavity. The results of this investigation show that the presence of the cylinder results in an increase in the average Nusselt number compared with a case with no cylinder. This result is observed for both, an adiabatic and isothermal, boundary condition imposed on the cylinder. The average Nusselt number increases with an increase in the Richardson number for all non-dimensional radius of the cylinder studied in this work. Moreover, the optimal heat transfer results are obtained when placing the cylinder near the bottom wall for various Richardson numbers. For dominant natural convection ($Ri \geqslant 2.5$), the average Nusselt number increases with an increase in the non-dimensional radius for $0.05 < r_o/H < 0.2$. Further increase in the non-dimensional radius does not change the Nusselt number at a particular *Ri*. For dominant mixed convection, the average Nusselt number increases with an increase in the radius of the cylinder for various Richardson numbers.

1. Introduction

Mixed convection in enclosures with a moving boundary is an interesting problem largely owing to the enhancement in the flow mixing and recirculation on account of the shear stress generated by the moving surface, and its overall effect on the imposed thermal gradient. This phenomenon occurs in a number of engineering applications from solar power, microelectronics to nuclear reactors [1–7]. In the mixed convection mode of heat transfer, features of both forced and natural convection are important. In addition to the numerous engineering applications, investigation of various configurations under which mixed convection heat transfer occurs, is of interest as fundamental problems in heat-transfer and fluid flow. The impact of parameters such as Rayleigh number, Richardson number and Grashof number (*Ri* has effect of *Re* on it) on the overall Nusselt number can lead to better understanding and newer engineering applications requiring enhanced heat and mass transfer. For instance, turbulent mixed-convection heat transfer in vertical pipes is of interest to sodium cooled reactors [8] or enhanced air-conditioning for improved indoor air quality [9].

Introduction of an obstruction in an enclosure impacts the flow field and heat transfer. Several studies dealing with enhancing natural convection in obstructed cavities are documented in the literature [10–13]. Literature on the body inserted lid-driven cavity is sparse [14–24]. Dagtekin and Oztop [14] inserted an isothermally heated rectangular block in a lid-driven cavity at different positions to simulate the cooling of electronic equipments. They found that dimension of the body was the most effective parameter on mixed convection flow. MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block was studied by Rahman et al. [15–18]. The numerical results indicated that the Hartmann number, Reynolds number and Richardson number had strong influence on the streamlines, isotherms, average Nusselt number at the hot wall and average temperature of the fluid in the enclosure. Islam et al. [22] studied mixed convection in a lid driven cavity with an isothermally heated square blockage and isothermal cold wall temperature imposed on all cavity walls. The top wall of the cavity was assumed to move rightward with a constant speed. Their results showed that Richardson number, blockage ratio and blockage placement eccentricities have an effect on the average Nusselt number measured

* Corresponding author at: Frankel Vascular Mechanics Laboratory, Biomedical Engineering Department, University of Michigan, Ann Arbor, MI 48109, USA. Tel.: +1 734 647 8033.

E-mail address: khanafer@umich.edu (K. Khanafer).

0017-9310/$ - see front matter
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.07.023

Nomenclature

g	gravitational acceleration
Gr	Grashof Number = $g\beta(T_h - T_C)H^3/\nu^2$
H	cavity side length
Nu	Nusselt number
P	dimensionless fluid pressure $= p/(\rho u_o^2)$
Pr	Prandtl number = ν/α
Re	Reynolds number = $u_o H/\nu$
Ri	Richardson number = Gr/Re^2
r_o	radius of the cylinder
r_o/H	non-dimensional radius of the cylinder
T	temperature
t	time
u	velocity in x-direction
u_o	lid speed
U	dimensionless horizontal velocity = u/u_o
v	velocity in y-direction
V	dimensionless vertical velocity = v/u_o
x, y	cartesian coordinates
X, Y	dimensionless cartesian coordinates, $(x, y)/H$

Greek symbols

α	thermal diffusivity of the fluid
β	coefficient of thermal expansion of fluid
δ_x, δ_y	dimensionless coordinates of the porous block
ν	kinematic viscosity
θ	dimensionless temperature = $(T - T_C)/(T_h - T_C)$
ρ	density
Ψ	dimensionless stream function

Subscripts

C	cold
h	hot

along the solid block. Oztop et al. [23] studied mixed convection heat transfer characteristics in a lid-driven differentially heated cavity having a circular body. In that study, flow was driven by the left hot wall moving upward with a constant speed while the other walls were assumed stationary. The horizontal walls were assumed adiabatic while the right wall was maintained at a lower temperature than the left wall. Three different boundary conditions were imposed on the inner cylinder, namely, adiabatic, isothermal or conductive. Their results showed that the direction of the moving lid was found to be the most important parameter on flow and temperature characteristics within the cavity. Mixed convection heat transfer in a lid-driven cavity along with a heated circular hollow cylinder positioned at the center of the cavity was analyzed numerically by Billah et al. [24]. The left cold wall was assumed to move upward with a constant speed. The horizontal walls of the cavity were adiabatic while the right wall was maintained at a cold temperature. This study showed that the flow field and temperature distribution were strongly depending on the cylinder diameter and the solid–fluid thermal conductivity ratio.

Earlier studies on mixed convection in lid-driven enclosures with an obstacle considered either heated obstacles or a differentially heated cavity. Many engineering applications such as chemical reactors, mixing tanks and boilers have obstructions (such as stirrers with blades) wherein the lower surface of the reactor/boiler is at a higher temperature as compared to the upper surface. The heat and mass transfer characteristics of such engineering applications are difficult to evaluate from a numerical standpoint. However, simplified configurations can provide useful engineering insights and scaling information. With this goal in mind, the current numerical investigation aims at exploring the effects of the Richardson number, size and location of a circular obstruction on the momentum and energy transport processes in the lid-driven cavity heated from below. Two temperature boundary conditions on the cylinder were investigated, namely, adiabatic and constant temperature. The results of this investigation are presented in terms of streamlines, isotherms, and average Nusselt number at the hot surface of the cavity.

2. Mathematical formulation

The physical system under study with the system of coordinates is depicted in Fig. 1. The flow inside the cavity with height H is considered steady, laminar, incompressible, and two-dimensional. A circular cylinder of radius r_o (=0.2H) is inserted within the cavity and placed at nine prescribed dimensionless locations (δ_x, δ_y) with the default case being at the center of the cavity $(0.5H, 0.5H)$. No slip boundary condition is imposed on all cavity walls and cylinder surfaces. The top wall is moving towards the right with a constant speed u_o. The vertical walls of the cavity are considered adiabatic while the top wall is maintained at a colder temperature T_C than the bottom wall, which is maintained at T_h. Different thermal boundary conditions for the circular body are imposed, namely, adiabatic or isothermal (T_C). Moreover, the cavity is filled with air (Pr = 0.7) as the working fluid and the thermophysical properties of the working fluid are taken to be constant except for the density variation, which is assumed to vary linearly with temperature according to the Boussinesq approximation.

Based on the above considerations, the following variables are introduced to render the analysis dimensionless

$$X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{u_o}, \quad V = \frac{v}{u_o}$$
$$\theta = \frac{(T - T_C)}{(T_h - T_C)}, \quad P = \frac{p}{\rho u_o^2} \tag{1}$$
$$Re = \frac{u_o H}{\nu}, \quad Gr = \frac{g\beta(T_h - T_C)H^3}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}$$

where P is the dimensionless pressure, Re the Reynolds number, Gr the Grashof number and Pr the Prandtl number. According to the above assumptions and dimensionless variables, the normalized

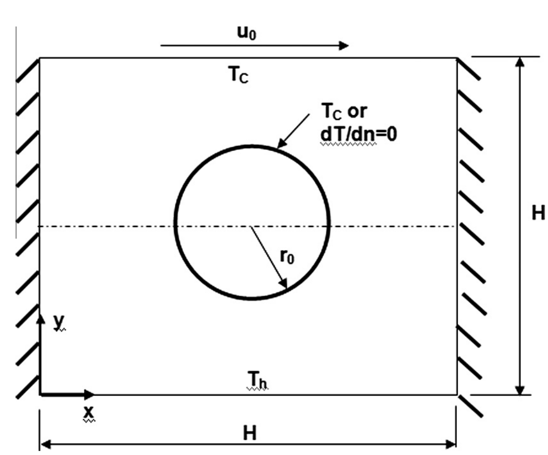

Fig. 1. Schematic of the model with coordinates and boundary conditions.