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Thermal analysis and design of a multi-layered rigidity tunable composite

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ABSTRACT

Elastomer-based composites embedded with thermally-responsive material (TRM) and a liquid-phase Joule heater are capable of reversibly changing their elastic rigidity by up to four orders of magnitude. At room temperature, the TRM layer is rigid and prevents the surrounding elastomer from elastically bending or stretching. When activated, the embedded Joule heater softens or melts the TRM, which leads to a dramatic reduction in the elastic rigidity of the composite. In this manuscript, we examine the activation of these composites by performing analytical, numerical, and experimental studies of the temperature distribution, thermal history, and phase transition. We consider both low melting point (LMP) metal alloys (e.g. Field's metal) and shape memory polymer (SMP). An analytical solution using the Galerkin Based Integral (GBI) method is derived for the cases where no phase change is involved, while a numerical scheme using the Latent Heat Accumulation (LHA) method is utilized to probe scenarios where phase change has a central role in the elastic rigidity change. The analytical and numerical studies predict a temperature history that is in good agreement with experimental measurements obtained with an IR thermometer. Analysis of the internal temperature distribution leads to scaling laws for determining the required activation time and allowable input power rate for composites containing either LMP alloys or SMP. These scaling laws could potentially be used to inform the design of rigidity tunable composites (RTC) used in assistive wearable technologies and biologically-inspired soft-matter robotics.

1. Introduction

Inspired by natural muscle and the catch connective tissue in echinoderms such as sea cucumbers [1], rigidity tunable materials represent an exciting new area in the emerging field of biologically-inspired robotics. These rigidity tunable materials have potential applications in both the military for injury prevention and civilian realms such as wearable robotic assistive devices [2]. Previous efforts for rigidity tunable multifunctional materials include using chemically tuned nano-composite [1], magneto-rheological fluids [3,4], and pneumatic particle jamming [5,6]. While promising for rigidity control in conventional machines and robotics, these methods require external pneumatic, fluidic, and mechanical hardware that may limit their functionality in soft or miniaturized host platforms.

In recent years, engineers have demonstrated reversible elastic rigidity tuning with shape memory polymer-steel laminate composites that are activated with external heating [7,8]. The shape memory polymer exhibits variable stiffness and functions as a connective medium that controls the relative displacement of the rigid components. At lower temperature the laminate has high flexural rigidity. Upon external heating, however, the SMP softens and allows for translation of the rigid components during deformation. The resulting change in rigidity can be as large as two orders of magnitude. This approach still requires external heating, but the laminate hybrid composite concept opens up new opportunities for design of multifunctional materials with tunable rigidity.

Based on previous efforts, including the recent development of masked deposition techniques for elastically soft electronics [9,10], most recently Shan et al. have fabricated rigidity tunable composites (RTC) with an on-board soft heater [2]. These composites are composed of multiple layers of acrylic VHB tape with low melting point (LPM) metal alloys, such as Field's metal, or with shape memory polymer (SMP). This prototype demonstrates rigidity changes of up to four orders of magnitude [2]. The integrated soft heater component allows the RTC to be used in applications such as wearable assistive devices that require on-board functionality and low power input.

Although promising, the actuation mechanism and underlying physics for RTC technologies with embedded soft-matter heaters has yet to be adequately examined. The presence of phase change and heterogeneity between composite layers presents modeling challenges. There have been recent efforts on analytical and

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Nomenclature

$\mathbf{A},\mathbf{B},\mathbf{D},\mathbf{P}$	matrices
a	thermal diffusivity [m^2/s]
b,d,L	dimensions of samples [m]
C_p	specific heat [J/kg/K]
$\tilde{\mathbf{d}}_n$	eigenvectors
$f_i(x)$	basis function
$g(x,t)$	heat source intensity [$J/m^3/s$]
h	heat transfer coefficient [$W/m^2/K$]
I	current [A]
P	power [W]
q	heat flow rate [W]
R	electrical resistance [Ω]
$T(x,t)$	temperature [°C]

Greeks

ℓ	specific latent heat [J/kg]
ρ	density [kg/m^3]
κ	thermal conductivity [W/m/K]
γ_n	eigenvalues
δ_i,β_i,η_i	coeffcients
$\theta(x,t)$	differential temperature [°C]

Abbreviations

TRM	thermally-responsive materials
GBI	Galerkin Based Integral
HBI	heat balance integral
SMP	shape memory polymer
LHA	latent heat accumulation
RTC	rigidity tunable composite
LMP	low melting-point

Subscripts

G	Galinstan
V	VHB
F	Field's metal
S	SMP
air	air related variables
g	glass transition temperature
melt	melting point
ef	effective value
e	inclusions, Galinstan, or Field's metal, or SMP

numerical modeling of phase change in 1-D infinite domains [11–15]. There have also been heat diffusion modeling in composites containing heterogeneous materials [16–21]. The combination of these efforts may provide solutions for modeling the current RTC composite design.

Typical analytical approaches to solve a heat diffusion equation include separation of variables, Green's function method, the Heat Balance Integral (HBI) method, and the Galerkin Based Integral (GBI) method [20,22-24]. Here, because of the relatively complex geometry and heterogenous materials properties, an exact closed-form analytical solution is not readily available. Thus we turn to the integral methods for an approximated solution that is closed-form [22].

The HBI method was first used by von Karman and Pohlhausen to solve boundary-layer problem in fluid mechanics [25]. Goodman then introduced its use in a one dimensional melting problem [26]. Since then this method has found many applications in the solution of one-dimensional heat transfer problems including the one-phase Stefan problem and linear hyperbolic heat-conductions problems on a semi-infinite medium [12-15]. However, these studies cannot be easily extended to a multi-layered problem involving heat generation and phase change within finite domains. The Galerkin based integral method used by Haji-Sheikh et al., which successfully handles heterogeneous material properties [16–18], is more naturally suitable for the problem at hand.

The most popular numerical approaches to one dimensional heat diffusion problems involving phase changes are the enthalpy methods [27–29,11], including many variants such as the apparent heat capacity method [29], and the latent heat accumulation (LHA) method [11]. These methods typically avoid the explicit tracking of the phase change front and thus are easier to implement and require less computational cost. The apparent heat capacity method introduces an enormously large heat capacity over a very small region around the phase change temperature to account for the latent heat absorbed during phase change [11]. The LHA method based on finite volume formulation, however, not only provides the flexibility to track the melting front but also ensures the conservation of energy over the whole domain [11].

In this manuscript, we perform a comprehensive study to examine the activation of RTCs embedded with liquid metal alloys and elastomers. The purpose of this paper is to identify the scaling laws that govern the activation times and power requirements for this new class of rigidity-tunable composite. First, an analytical model based on the GBI approach is established and used for rigidity tuning without phase change. Next a one dimensional finite volume numerical model is developed based on the LHA approach to simulate cases involving a constant-temperature melting process. The predicted surface temperature of the composite is then compared with experimental measurements and provides feedback to the numerics. After confirming the validity of the models, temperature profile within the composite are predicted and guidelines on material choice for the RTC are discussed.

2. Material and methods

The RTCs used in this study are made of multiple layers of acrylic elastomer films (VHB™ tapes, 3M™) embedded with a soft-matter Joule heater and layer of thermally-responsive material (TRM). The Joule heater is composed of a serpentine channel of Gallium-Indium alloy (Galinstan; RotoMetals, Inc.) that is liquid at room temperature. Because the heating element is liquid, it can elastically deform with the surrounding material once the composite is activated. The TRM layer may be composed of either a low melting point (LMP) metal alloy (in this study, we used Field's Metal with melting point T_{melt} = 62 °C; RotoMetals, Inc., Fig. 1(a)) or shape memory polymer composites (SMP, Veritex™ with activation temperature T_g = 62 °C, Cornerstone Research Group, Inc., Fig. 1(b)).

Details of the rapid fabrication method used to produce these samples are presented in Ref. [2]. In summary, the RTC is composed of two functional components, the Joule heater and the TRM layer (Fig. 1(d)). For the Joule heater, a serpentine channel of Galinstan is deposited on a layer of VHB tape using a laser-patterned mask (30 Watt CO_2 laser engraver; VLS 3.5; Universal Laser Systems, Fig. 1(c)). In the case of Field's metal, the TRM is prepared by depositing melted LMP alloy on a masked layer of VHB tape and then using the double-stick adhesion of the VHB tape to bond it to the Joule heater.