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Heat/mass transport in a drop translating in time-periodic electric fields

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ABSTRACT

Enhancement of heat or mass transport in a spherical drop of a dielectric fluid translating in another dielectric fluid in the presence of uniform and non-uniform time periodic electric fields is investigated. The internal problem or the limit of the majority of the transport resistance being in the dispersed phase is considered. The transient energy conservation equation is solved using a fully implicit finite volume method. Lagrangian tracking of fluid particles is carried out to understand the extent of fluid mixing and heat/mass transport inside the drop. The effect of electric field is expressed in terms of L, the ratio of the maximum electric-field-induced surface velocity to translation-induced surface velocity. For a fixed value of L, compared to a steady uniform field, unsteady electric fields are more effective in enhancing heat transfer due to fluid mixing resulting from changing flow patterns. The enhancement in time periodic fields shows a non-monotonic dependence on electric field frequency. For low frequency, the time scale of electric field-induced mixing is large and hence transport enhancement is low. For very high frequency, the field changes before fluid particles have moved any significant distance which limits fluid mixing and the heat transport behavior remains essentially that of a purely translating drop. Therefore the optimum frequency that corresponds to the maximum transport enhancement lies between these two limits. Furthermore, with periodic fields for a fixed L, increase in Peclet number always increases the maximum heat transfer enhancement. However, for a fixed Peclet number, an increase in the electric field strength does not necessarily lead to an increase in heat transfer enhancement. Higher electric field strength leads to higher flow velocity (which facilitates heat transfer) but provides shorter time scale over which fluid mixing takes place (which is detrimental to heat transfer). These competing effects give rise to non-monotonic enhancement behavior with L. For unsteady non-uniform electric field, heat/mass transport is not enhanced when $L < \sim 0.5$. For larger values of L, non-uniform electric field provides higher transport enhancement compared to the steady and unsteady uniform electric fields.

1. Introduction

When a fluid droplet is translating in another continuous medium fluid under the effect of buoyancy or gravity force in steady Stokes flow, a circulatory vortex known as the Hill's spherical vortex develops inside the drop. Hadamard and Rybczynski were the first to report the solution for the flow field inside and outside the drop. Using the Hadamard–Rybczynski flow, mass/heat transfer to and from the drop has been investigated in two limits based on the relative magnitude of the transport resistance in the dispersed and in the continuous phase. The internal problem, where the transport resistance is mainly inside the drop, was considered by Kronig and Brink [1]. When the drop surface temperature is suddenly changed, they showed that the internal Nusselt number oscillates due to the motion of the fluid from the drop interior to the drop surface. The temperature eventually equilibrates along streamlines and the Nusselt number attains a steady value. At $\mathrm{Pe} \to \infty$, the temperature contours coincide with the streamlines and the heat transfer is dominated by diffusion across the streamlines. The increase in Nusselt number is due to the decrease in the distance over which conduction takes place and the Nusselt number becomes 17.66 which is about 2.7 times the value for pure diffusion ($2\pi^2/3$).

The external problem is the other limiting case where the resistance to mass/heat transfer from the drop surface is primarily in the continuous phase. The Hadamard and Rybczynski flow field in the continuous phase is the first-order solution in the Reynolds number. Taylor and Acrivos [2] provided the solution with higher order correction using singular perturbation technique, and later derived the steady state Nusselt number as a function of the Peclet number. Both limits were considered by Abramzon and Borde [3] who addressed the conjugate problem of heat transfer to a translating drop when the resistance to heat transfer in both phases is comparable and showed that the overall Nusselt number can be

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Nomenclature

a	drop radius
A_s	drop surface area
E	electric field
E_0	amplitude of the uniform electric field
k	thermal conductivity
L	ratio of the maximum surface velocity due to a steady uniform electric field to that induced by drop translation, Eq. (6)
Nu	Nusselt number
Pe	Peclet number based on drop translation velocity ($\mathrm{Pe} = 2U_\infty a/\alpha$)
q	net rate of heat transfer
R	electrical conductivity ratio (σ/σ_{in})
(r, θ,)	spherical coordinates
S	electrical permittivity ratio ($\varepsilon/\varepsilon_{in}$)
t	time
T	temperature
u	velocity
U	maximum surface velocity due to steady uniform electric field, Eq. (7)
U_{mig}	drop migration velocity, Eq. (5)
U_∞	pure translation velocity
U_{net}	drop net translation velocity, Eq. (22)
x	position vector
z	transformed coordinate

Greek symbols

α	thermal diffusivity
β	relative importance factor, Eq. (2)
ε	electrical permittivity
η	heat transfer enhancement factor, Eq. (20)
σ	electrical conductivity
μ	viscosity
λ	viscosity ratio (μ_{in}/μ)
ω	electric field frequency
ψ	Stream function

Subscripts

b	bulk
g	straining
el	induced by electric field
in	inside the drop (dispersed phase)
s	surface
tr	translation
u	uniform

Superscript

$*$	dimensionless

calculated as $1/\mathrm{Nu} = k/k_{in} \times 1/\mathrm{Nu}_{in} + 1/\mathrm{Nu}_{ext}$. For drops moving at intermediate and higher Reynolds numbers a large body of experimental, analytical, and computational work on mass/heat transfer from a drop is available in the literature. Review of this work is beyond the scope of this paper and readers are referred to authoritative monographs by Clift et al. [4], Sadhal et al. [5], and Sirignano [6].

Application of an electric field to a drop of a dielectric fluid in another immiscible fluid induces flow both inside and outside the drop [7]. Taylor [7] used a leaky dielectric model to calculate the electric field-induced stresses at the drop surface and derived the stream functions for flow inside and outside the drop in Stokes flow regime. Oliver, Carleson, and Chung [8] investigated transient heat transfer to a drop suspended in steady uniform electric field when the drop surface temperature is suddenly changed and the bulk of the thermal resistance is considered in the drop. At high Peclet numbers, the maximum steady state Nusselt number reaches a value of 29.8 [9]. This value is independent of the Peclet number and it is 67% higher than the maximum theoretical steady state Nusselt number for a translating drop. The effect of drop deformation on heat transfer to a suspended drop was computationally analyzed by Hader and Jog [10]. They found that for oblate drop the enhancement in heat transfer is more than that for a prolate drop for both the internal and the external problems. For a drop translating in a uniform electric field, the steady state Nusselt number varies from that for translation without electric field to that for pure electric field driven flow (without translation) (Chang et al. [11], Chung and Oliver [12], Nguyen and Chung [13]).

Homsy and co-workers [14–17] (Ward and Homsy [14,15], Christov and Homsy [16], Xu and Homsy [17]) considered the problem of mass transport inside a translating drop that is subject to time-periodic electric field in Stokes flow regime. They showed that the resulting flow field inside the drop consists of two vortices of different sizes that are separated by a separation disk. As the electric field strength changes with time, the separation disk moves. The resulting flow field gives rise to chaotic advection in the droplet. Increase in the motion of the separation disk and an increase in the electrically-driven flow relative to translation, lead to a larger portion of the drop volume being mixed due to chaotic advection. The chaotic advection was experimentally demonstrated by Ward and Homsy [15]. Christov and Homsy [16] conducted a numerical study of mass transfer inside a translating drop that is subject to time-periodic electric field. It is known that the drop bulk temperature decays exponentially once Nusselt number reaches a constant value. The ratio of the decay constant to that for pure diffusion was defined as the mass/heat transfer enhancement factor [1]. For time-periodic electric field, they found that the enhancement factor exhibits spectral resonant peaks at several values of frequency. Lee et al. [18] investigated mass transfer enhancement for a suspended droplet subjected to a time periodic non-uniform electric field. They have reported that heat/mass transfer is enhanced due to occurrence of fluid mixing arising from dielectrophoretic migration of the drop. Also, their results showed that there is an optimal frequency at which transport is most enhanced. Recently, we considered heat/mass transport inside a suspended drop that is exposed to time periodic uniform and non-uniform electric fields [19]. It was found that the enhancement in heat/mass transfer is a non-monotonic function of the electric frequency and at high Peclet numbers the maximum heat (mass) transfer enhancement is obtained when the dimensionless electric field frequency is of the order of Peclet number. By tracking Lagrangian fluid particles, it was revealed that the application of non-uniform unsteady electric field gives rise to chaotic advection at high Pe whereas steady and unsteady uniform electric fields do not.

In this paper, we consider a translating drop in uniform and non-uniform time periodic electric field. We note that the uniform time periodic field of the form $E = E_0 \cos(\omega t)$ that is considered here is quite different from the field that was been investigated by Christov and Homsy [16]. The amplitude of their electric field never goes zero in a cycle and the stagnation disk moves only a short distance