International Journal of Heat and Mass Transfer 66 (2013) 355–365

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Numerical and experimental study of steady state free convection generated by constant heat flux in tilted hemispherical cavities

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ARTICLE INFO

Article history:
Received 16 June 2013
Received in revised form 1 July 2013
Accepted 10 July 2013
Available online 7 August 2013

Keywords:
Heat transfer
Natural convection
Tilted hemispherical cavities
High Rayleigh numbers
Experimental approach
CFD
Engineering applications

ABSTRACT

Steady-state natural convection taking place in hemispherical air-filled cavities is presented in this work. The circular base of the hemispherical cavity is the hot active wall subjected to a constant heat flux. The closing dome is maintained isothermal at a lower temperature and acts as a cold wall. This cavity can be inclined at an angle α between 0° (hot wall horizontal) and 90° (hot wall vertical) in steps of 15°. The problem is studied both experimentally and numerically. Measurements of thermal variables are taken on an experimental assembly at steady state in order to characterize the heat exchanges at the hot wall. The Rayleigh number resulting from the experimental parameters varies between 3.44×10^5 and 2.83×10^7. The numerical study covers a larger *Ra* range, between 10^4 and 5×10^7. Calculations performed using the finite volume method complement the experimental results by examining also the dynamic aspects. The mathematical model used is validated by measurements. Differences between the two approaches are found to be relatively low, always within the uncertainties of the experimental data. The comparison with previous published works dealing with the horizontal cavity is also satisfactory. The resulting thermal and dynamic fields are presented for all the angles treated. Correlations between average Nusselt and Rayleigh numbers are proposed to quantify the convective exchanges for engineering applications.

1. Introduction

Natural convection in closed cavities has been extensively treated in recent years, showing a great interest for engineering. The fluid flow in such cavities depends on several physical parameters, playing the geometry of the enclosure a central role. Several geometries have been considered depending on what is the proposed application. The hemispherical shape has been the object of some works in nuclear engineering. Security is of paramount importance in this area, as it is necessary to fully control the thermal state of all the elements of a plant in operation. It is also necessary to study the scenarios of possible plant failures which could lead to the thermal runaway or to important accidents. The study of Shiina et al. [1] focused on the heat transfer by natural convection that occurred in a hemispherical cavity formed by a hot horizontal circular base and a cooled covering dome. The flows were visualized for Rayleigh numbers reaching up to 3×10^7 based on the radius base and for several Prandtl numbers *Pr* between 6 and 860. The authors proposed correlations of the Prandtl–Nusselt–Rayleigh type that are reduced to Nusselt–Rayleigh correlations because they showed that the influence of *Pr* on heat transfer is small in this case. In [2], Shiina et al. treated the same hemispherical cavity for a wider range of Rayleigh number and values of Prandtl number *Pr* ranging from 6 to 13,000. They confirmed that the influence of *Pr* is still negligible and proposed new Nusselt–Rayleigh correlations for the ranges 10^6–10^9 and 10^9–6×10^{10}. In this numerical study, where the temperature of the disk is imposed (Dirichlet-type boundary condition), the authors distinguish a steady circulating flow for Rayleigh numbers $\leqslant 10^6$. For higher values of this number, a flow characterized by periodic thermal plumes appears and finally the regime becomes turbulent for Rayleigh numbers $\approx 10^9$. The calculation of such Rayleigh number is based on the temperature difference between the disk and the dome. This definition is different from that considered in the present work, where a condition of imposed heat flux (Neumann-type boundary condition) is adopted. The influence of the Prandtl number is also discussed among others in the book by Bird [3], with an interesting dimensionless approach. Relationships between Nusselt and Rayleigh numbers have also been proposed by Lewandowski et al. for isothermal horizontal [4] and vertical [5] hemispheres in the case of open domains. Rossby includes in [6] relations for the case of closed cavities. Lee et al. [7] simulated the heat transfer that would occur in the case of a

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.07.038

Nomenclature

a	thermal diffusivity of the air
C_p	specific heat at constant pressure
g	acceleration due to the gravity
g_r, g_ϕ, g_θ	components of the acceleration due to the gravity in r, ϕ, θ directions respectively
$\overline{\overline{h_{m,\alpha}}}$	steady state average measured convective coefficient at the hot wall for an angle α
I_u,I_g	current intensities through the electrical resistances of the useful and guard zones respectively
k	coefficient of the correlations in Eq. (21)
n	exponent of Ra number in Eq. (21)
n_o	outgoing normal to the surface of the base
$\overline{\overline{Nu}}$	steady state average Nusselt number at the hemisphere from other reference
$\overline{\overline{Nu_{c,\alpha}}}$	calculated steady state average Nusselt number at the hot wall for an angle α
$\overline{\overline{Nu_{m,\alpha}}}$	measured steady state average Nusselt number at the hot wall for an angle α
p	pressure
p^*	dimensionless pressure
P	power exchanged by convection
P_{cond}	power exchanged by conduction
$P_{cond,i}$	power exchanged by conduction for the ith element of the hot wall
P_{rad}	power exchanged by radiation
$P_{rad,i}$	power exchanged by radiation by the ith element of the hot wall
Pr	Prandtl number
$P_{t,g}$	total power dissipated by the guard area of the hot wall
$P_{t,u}$, P_t	total power dissipated by the useful area of the hot wall
r	radial co-ordinate
r^*	dimensionless radial co-ordinate
r_u,r_g	electrical resistances of the useful and guard zones of the base respectively
R	radius of the hemispherical cavity
Ra	Rayleigh number adopted in this work defined by Eq. (7)
Ra_T	Rayleigh number for Dirichlet-type condition $Ra_T = \frac{g\beta R^3 \rho}{\mu a}(T_h - T_c)$
S_h	area of the hot wall
S_i	area of the ith element of the hot wall
S_u, S_g	area of the useful and guard zone respectively
T	local temperature
T^*	dimensionless temperature
T_c	temperature of the dome (cold wall)
T_e	temperature of the external ambiance
T_h	local temperature of the base (hot wall)
$\overline{T_c}$, $\overline{T_h}$	average measured temperature of the cold and hot walls respectively
T_0	initial uniform temperature of the whole system
u_r, u_ϕ, u_θ	flow velocity components in r, ϕ, θ directions respectively
$\vec{u}$	flow velocity vector
$\vec{u}^*$	dimensionless flow velocity vector
x, y, z	Cartesian coordinates (reference frame tied to the inclined hemisphere)
x', y', z'	Cartesian coordinates (fixed reference frame)

Greek symbols

α	inclination angle of the cavity
β	volumetric expansion coefficient of the air
ΔG	experimental absolute error of the generic parameter G
ξ	radiation exchange factor
ε_h	global infrared emissivity of the inner face of the hot wall
ε_c	global infrared emissivity of the inner face of the cold wall
$\varphi_{t,g}$	total heat flux dissipated by the guard zone
$\varphi_{t,u}$	total heat flux dissipated by the useful zone
φ	total heat flux dissipated by the hot wall
ϕ, θ	angular spherical coordinates
λ	thermal conductivity of the air
λ_i	thermal conductivity of the insulation material
μ	dynamic viscosity of the air
ρ	density of the air

severe accident in nuclear power plants. In this case, the molten core can relocate to the lower plenum of the reactor vessel and form a hemispherical pool. This work also involved a hemispherical cavity consisting of a horizontal disk with the dome below and a volumetrically heated fluid. They showed, among other results that (i) the time for steady state for water is much greater than for air, by a factor of about 4, (ii) the upper heat flux ratio is about unity at all locations in the water and air tests (iii) heat transfer increases with augmentation of the pool angle, except in the vicinity of the disk. Their results are consistent with other studies in which Nusselt–Rayleigh correlations are proposed as those of Lee and Suh [8] Gabor et al. [9], Asfia and Dhir [10] and Theofanous and Angelini [11]. The work by Kelkar et al. [12] contains a review of correlations for this geometry. Khubeiz et al. [13] treated the same geometry and proposed correlations from numerical and experimental work. Laouadi and Atif [14] studied numerically, by means of the control volume method, the heat exchanges that occur in steady state laminar natural convection within multi-layer domes with uniform spacing heated from outside. They proposed correlations in which several influencing parameters are included. In a recent study, Saber and Laouadi [15] presented the 2D convective flows that take place in this type of cavities under different boundary conditions.

The case of hemispherical cavities where the circular base (disk) is inclined with respect to the horizontal plane has been very scarcely reported in the literature. This is probably due to the complexity of numerical and experimental approaches that require significant resources for this geometry and that can only be tackled in 3D. The numerical study of Cabelli [16], which is among the first ones dealing with this geometry, contains a detailed characterization of flows that occur in steady state laminar conditions. The explanations about the influence of the inclination angle on the dynamic aspects and those related to the simulated particle tracks are very illustrative. Once more, the case considered is that of imposed temperature on the active disk (Dirichlet-type boundary condition) but with the central part of the disk at a different temperature. The study of Cabelli does not include the quantification of the convective heat fluxes neither for the horizontal disk nor for the inclined cavities.

The purpose of the present work is to obtain Nusselt–Rayleigh correlations that can be useful for engineering applications where inclined hemispheres are employed. In this work, the real hot wall consists of an electronic circuit connected to a power supply. It is the main element of the assembly and can be placed at different positions relative to the horizontal plane. Besides the experimental measurements the study includes numerical simulations by means