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A-priori study of subgrid-scale models for the flow field in the rotor exit region of a centrifugal turbomachine

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ABSTRACT

The present article deals with a-priori evaluation of two popular subgrid-scale (SGS) models, i.e. Smagorinsky and similarity, for the complicated flow field of a centrifugal turbomachine. Comparison of Smagorinsky and similarity models for various turbulent flows shows that accuracy of these models is inferior for complicated turbomachinery flow. The estimated SGS model coefficients, correlation coefficient among exact and modeled SGS quantities and the functionality between SGS stress/dissipation and resolved flow parameters are different features of SGS models which are examined for Smagorinsky and similarity models. The calculated model coefficients for the rotor exit flow are significantly smaller than their classical values to avoid over-estimation of SGS dissipation. Back-scattering of turbulent energy and spectral shortcut mechanisms are two possible reasons for this reduction of models coefficients. Investigation of instantaneous SGS dissipation shows that about 40% of flow samples in the rotor exit region have back-scattering of energy. This large density of back-scattering significantly reduces the performance of fully-dissipative models such as the Smagorinsky model. Joint probability density functions of exact vs. modeled SGS stress/dissipation show that the similarity model is capable of back-scattering prediction and has a considerably larger correlation coefficient than that of the Smagorinsky model. The present article shows that the Smagorinsky model performance improves in the presence of straining (in the jet–wake interacting regions) while the minimum correlation coefficient occurs in the core region of jets and wakes with smallest straining. Weak functionalities between Smagorinsky-modeled SGS stress/dissipation with the resolved strain rate tensor, particularly in the case of cross components, show the necessity for modifying Smagorinsky model in such a complicated flow field to allow for spectral shortcut and energy back-scattering mechanisms.

1. Introduction

Turbulent flow in the rotor exit region of a turbomachine is complicated, three dimensional and anisotropic. This flow field consists of a diverse range of time and length scales, ranging from Kolmogorov to integral scales. Investigation and modeling of complex fluid dynamics phenomena in such flow regimes requires sufficient knowledge about the unsteady flow field.

Large eddy simulation (LES) is a numerical tool for simulating the unsteady flow fields with high Reynolds numbers where flow variables are decomposed into resolved (large) and unresolved or subgrid (small) scales. The decomposition in LES is performed by filtering the velocity field, $\mathbf{U}(\mathbf{x},t)$ [1]:

$$\overline{\mathbf{U}}(\mathbf{x},t) = \int_D G_\Delta(\mathbf{r},\mathbf{x})\mathbf{U}(\mathbf{x}-\mathbf{r},t)d\mathbf{r} \tag{1}$$

where D is the computational domain, G_Δ is the filtering kernel with the characteristic scale Δ and the over-bar denotes filtering operation. The filtered continuity and Navier–Stokes equations for incompressible flows are:

$$\frac{\partial \overline{u_i}}{\partial x_i} = 0 \tag{2}$$

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j}\frac{\partial \overline{u_i}}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(\frac{\bar{p}}{\rho}\delta_{ij} + \tau_{ij}\right) + \nu\frac{\partial^2 \overline{u_i}}{\partial x_j^2} \tag{3}$$

where the subgrid-scale stress, τ_{ij}, is defined as:

$$\tau_{ij} = \overline{u_i u_j} - \overline{u_i}\,\overline{u_j} \tag{4}$$

The momentum equations must be closed by expressing the subgrid-scale stress tensor as a function of resolved variables. Resolved velocity field is directly simulated from the filtered Navier–Stokes equations and subgrid scales, that are more universal, are modeled [2]. Although an appropriate SGS model should accurately predict SGS stress, but in various subgrid-scale models, the predicted SGS

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Nomenclature

C	SGS model coefficient
D	computational domain
d_p	particle tracer diameter
G_Δ	filtering kernel with the characteristic scale Δ
p	pressure
r_{tip}	rotor outer radius
$\mathbf{r}$	spatial vector
S_{ij}	strain rate tensor components
$\|S\|$	characteristic strain rate (strain magnitude)
SG	specific gravity
t	time
u_i	velocity components, $i = 1, 2, 3$
$\mathbf{U}$	velocity vector
U_{tip}	tangential velocity at the rotor exit
x_i	Cartesian coordinates
$\mathbf{x}$	spatial vector
Greek Symbols	
δ_{ij}	Kronecker delta function
δ_x, δ_y	PIV grid spacing in x and y directions, respectively
δ	characteristic PIV grid spacing, $\delta = \sqrt{\delta_x \delta_y}$
Δ	filter length scale
η	Kolmogorov length scale
ν	kinematic viscosity
ν_t	eddy viscosity
Π	subgrid-scale dissipation
Π^-	backward energy transfer
Π^+	forward energy transfer
ρ	correlation coefficient; density
τ_{ij}	subgrid-scale stress tensor components
τ_f	Kolmogorov time scale
τ_p	particle response time
Other Symbols	
-	first filtering operation
$\sim$	second filtering operation
$\langle\cdot\rangle$	ensemble averaging
Scripts	
diss	dissipation-based
mod	model
sim	similarity model
Smag	Smagorinsky model
x, y, z	Cartesian coordinates

stress tensor deviates from the real stress tensor. The deviation is more significant for vortical and anisotropic flow fields such as those in turbomachines.

There are two approaches to evaluate a SGS model: (a) "*a-posteriori test*" and (b) "*a-priori test*". In a-posteriori test, large eddy simulation results are compared with results from direct numerical simulation (DNS) or experimental data. In this approach, a particular model is evaluated after its implementation in the simulation. The second approach, a-priori test, uses the results of DNS or experimental data and directly compares the modeled SGS stress tensor with the real SGS stress tensor. Available flow field data is used to calculate the real SGS stress from definition (Eq. (4)) and the modeled stress tensor from the model formulation. A-priori test based on experimental data is more feasible than a-posteriori test or DNS-based a-priori test, since the simulation of flow field requires large computational time, especially for complicated flow regimes. Among various experimental techniques in a-priori studies, particle image velocimetry (PIV) has the capability of measuring the instantaneous spatial velocity and is a good candidate for spatial analysis of flow fields and turbulence models.

1.1. Background on a-priori study of SGS models

1.1.1. Smagorinsky model

One of the most commonly used SGS models is the Smagorinsky model (Lilly hypothesis) [2,3]. According to this model, the deviatoric part of SGS stress tensor ($\tau_{ij} - \tau_{kk}\delta_{ij}/3$) is directly proportional to the resolved strain rate tensor $\left(\overline{S_{ij}}\right)$ and is expressed as

$$\tau_{ij}^{Smag} = -2\nu_t^{Smag}\overline{S_{ij}} \tag{5}$$

where the Smagorinsky eddy viscosity $\left(\nu_t^{Smag}\right)$ and $\overline{S_{ij}}$ are defined as

$$\nu_t^{Smag} = (C_{Smag}\Delta)^2|\overline{S}|, \tag{6}$$

$$\overline{S_{ij}} = \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right). \tag{7}$$

$|\overline{S}| = \sqrt{2\overline{S_{ij}}\,\overline{S_{ij}}}$ is the characteristic filtered strain rate (also known as strain magnitude) and C_{Smag} is the Smagorinsky coefficient. Since this coefficient is a constant, the Smagorinsky model expresses the SGS stress tensor as a function of the resolved strain rate tensor and the filter length scale. Therefore, the Smagorinsky coefficient is the only factor that justifies the accuracy of SGS turbulence quantities.

Although the Smagorinsky model is frequently used in simulations, this model has drawbacks.

- The model is one of eddy-viscosity type models which is completely dissipative and does not include energy transfer from subgrid-scale structures towards larger resolved structures (back-scatter of energy) [4]. It is additionally reported that the Smagorinsky model could over-predict SGS dissipation [5,6].
- The Smagorinsky coefficient should be adjusted for each flow field. The basis for Smagorinsky model is the proportionality between turbulent eddy viscosity and resolved velocity gradients [7]. Consequently, the Smagorinsky model over-estimates the turbulent viscosity if mean velocity gradients are considerable in comparison with turbulent ones. The coefficient should be zero near walls or in laminar flows [5]. A constant Smagorinsky coefficient in these regions over-estimates SGS stress and may prevent flow transition to turbulence [8]. In the case of near-wall turbulence, the integral length scale is in the order of filter size. The filter length scale is not an appropriate estimate for mixing length and the Lilly's hypothesis is not valid [7].
- Previous a-priori studies have proved that the correlation between modeled and real SGS quantities is small in various flow fields [4,9–11]. The inaccuracy in the prediction of SGS stress tensor could be due to non-alignment of subgrid-scale stress tensor and resolved strain rate tensor [10].

Investigation of 2 additional eddy-viscosity models based on vorticity and turbulent kinetic energy shows that no significant enhancement can be achieved by these models over the Smagorinsky prediction [10].

A-priori analysis of different flow fields have shown that the Smagorinsky coefficient strictly depends on flow regime and filter size, especially in straining and de-straining conditions [12]. Estimation of the Smagorinsky coefficient from balancing the modeled