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Fractional order heat conduction law in micropolar thermo-viscoelasticity with two temperatures

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ABSTRACT

Present work is concerned with the transient solution of a half-space problem in the context of fractional order micropolar thermo-viscoelasticity involving two temperatures whose surface is acted upon by a uniformly distributed thermal source. Medium is assumed initially quiescent. The formulation is applied to the fractional generalization of the Lord–Shulman theory with microstructure effects and the non-dimensional equations are handled by employing an analytical–numerical technique based on Laplace and Fourier transforms. The numerical estimates of the displacement, stresses and temperatures are computed for magnesium crystal like material and corresponding graphs are plotted to illustrate and compare theoretical results. All the fields are found to be significantly affected by the fractional parameter, viscosity and two temperature parameter. The phenomenon of finite speed of propagation is observed graphically for each field. Some particular cases have also been inferred from the present study.

1. Introduction

Under the assumption of continuum hypothesis of an elastic body, the classical theory of elasticity is based on linear stress–strain law (Hooke's law). In this theory, the transmission of load across a surface element of an elastic body is described by a force stress (force per unit area) and the motion is characterized by translational degrees of freedom only. For materials possessing granular structure, it is found that the classical theory of elasticity is inadequate to represent complete deformation. Certain discrepancies are observed between the results obtained experimentally and theoretically, particularly, in dynamical problems involving elastic vibrations of high frequencies and short wavelengths, i.e., vibrations due to the generation of ultrasonic waves. The reason for these discrepancies lies in the microstructure of the material, which exerts special influence at high frequencies and short wavelengths [1]. This influence of microstructure results in the development of new type of waves, not found in the classical theory of elasticity. Metals, polymers, composites, soils, rocks, concrete are typical media with microstructures. More generally, most of the natural and manmade materials including engineering, geological and biological media possess a microstructure.

Cosserat and Cosserat [2] were the first who rendered importance to the microstructure of a granular body and incorporated a local rotation of points in addition to the translation assumed in classical theory of elasticity. Consequently, there exists couple stress (a torque per unit area) in addition to the force stress. This theory is known as 'Cosserat theory' after their names or the theory of elasticity with couple stress. This theory was in dormant for so many years and did not get sufficient attention. In the 1960s, Eringen and Suhubi [3,4] and Eringen [5] gave modern formulations of Cosserat medium equations, which became known as the equations of the micropolar theory of elasticity or the theory of asymmetric elasticity. Within such a theory, solids can undergo macro-deformations and micro-rotations. The motion in this kind of solids is completely characterized by the displacement vector $\vec{u}(\vec{X},t)$ and the micro-rotation vector $\vec{\phi}(\vec{X},t)$ while in the case of classical elasticity, the motion is characterized by the displacement vector only. A historical development of the theory of micropolar elasticity is given in a recent monograph of Eringen [6]. This theory is expected to find applications in the treatment of mechanics of granular materials, composite fibrous materials and particularly microcracks and microfractures.

Recent years have seen an evergrowing interest in the investigation of dynamical interaction between thermal and mechanical fields in solids due to their manifold applications in various branches of engineering, science and technology. While in service, structural elements are frequently subjected to not only force loads but also non-uniform heating causing thermal stresses. These stresses themselves or in combination with mechanical stresses due to external loads may cause the material to fracture. Therefore, to perform a complete strength analysis of structures, it is necessary to know the magnitude and distribution of thermal stresses. In this connection, issues associated with the determination of

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Notations

σ_{ij}	components of force stress tensor
m_{ij}	components of couple stress tensor
$\vec{\phi}$	microrotation vector
λ^*	$\lambda_e(1+\alpha_0\frac{\partial}{\partial t})$
μ^*	$\mu_e(1+\alpha_1\frac{\partial}{\partial t})$
β_1^*	$\beta_{1e}(1+\beta_1\frac{\partial}{\partial t})$
β_{1e}	$(3\lambda_e+2\mu_e+k)\alpha_t$
β_1	$(3\lambda_e\alpha_0+2\mu_e\alpha_1)\frac{\alpha_t}{\beta_{1e}}$
$\theta = T - T_0$	thermodynamical temperature
$\phi = \phi - T_0$	conductive temperature
$\lambda_e,\ \mu_e$	Lame's constants
α_0, α_1	viscoelastic relaxation times
α, β, γ, k	micropolar material constants
α_t	coefficient of linear thermal expansion
a	two-temperature parameter
j	microinertia
T	absolute temperature
T_0	temperature of the medium in its natural state assumed to be $\lvert\theta/T_0\rvert \ll 1$
u_i	components of the displacement vector
ρ	density of the medium
e_{ij}	components of the strain tensor
e	cubical dilatation
c_E	specific heat at constant strain
K^*	thermal conductivity
τ_0	thermal relaxation time
Γ	Gamma function
m	fractional order parameter such that $0 < m \leqslant 1$

temperature fields and thermal stresses are of importance and draw the attention of experts of different professions. Keeping the above applications in view, the micropolar elasticity theory was further extended to include the thermal effects by Nowacki [7–9] and Eringen [10]. One can refer to Dhaliwal and Singh [11] for a review on the micropolar thermoelasticity and a historical survey of the subject as well as to Eringen and Kafadar [12] in the continuum physics series, in which the general theory of micromorphic media has been summed.

There has been very much written in recent years concerning the problem of propagation of thermal waves at finite speed. The articles of Dreyer and Struchtrup [13] and Caviglia et al. [14] provide an extensive survey of work on experiments involving the propagation of heat as a thermal wave. They reported instances where the phenomenon of second sound has been observed in several kinds of materials. Extensive reviews on the second sound theories can be found in the works of Chandrasekharaiah [15,16]. A generalized theory of linear micropolar thermoelasticity that admits the possibility of "second sound" effects was established in [17]. Using the Green and Lindsay theory [18], Dost and Tabarrok [19] propounded another new model of generalized micropolar thermoelasticity that permits the propagation of thermal waves at a finite speed. Chandrasekharaiah [20,21] obtained the equations for a generalization of micropolar thermoelasticity equations, which is called the heat flux dependent micropolar thermoelasticity and proved variational and reciprocal principles for his equations. Ciarletta [22] used the procedure proposed by Green and Naghdi [23] to derive a new linear theory of micropolar thermoelasticity. In this theory, in contrast to the theories developed in [17,19,20], the heat flow does not involve energy dissipation. Sherief et al. [24] proposed the generalized equations for the linear theory of micropolar thermoelasticity based on Lord–Shulman theory [25]. A uniqueness theorem is also provided in the same article. As an illustrative example, they have solved a half-space problem using Laplace and Hankel transforms whose boundary is rigidly fixed and subjected to an axisymmetric thermal shock. Several researchers in past including Kumar and his co-workers [26–28], Othman and Singh [29], Zakaria [30] among several others have studied many interesting problems based on micropolar generalized thermoelasticity theories.

Effect of internal friction on the propagation of plane waves in an elastic medium may be attributed to the fact that dissipation accompanies vibrations in solid media due to the conversion of elastic energy to heat energy. Several mathematical models have been used by authors [31,32] to accommodate the energy dissipation in vibrating solids where it is observed that internal friction produces attenuation and dispersion; hence, the effect of the viscoelastic nature of material medium in the process of wave propagation cannot be neglected. Also with the rapid development of polymer science and plastic industry as well as wide use of materials under high temperature in modern technology and application of biology and geology in engineering, the theoretical study and applications in viscoelastic materials have become an important task for solid mechanics. Further, as pointed out by Freudenthal [33], most of the solids, when subjected to dynamic loading, exhibit viscous effects. Keeping these facts in mind, several problems on wave propagation in a linear viscoelastic solid have been explored by many research workers. One can refer to Ilioushin and Pobedria [34] for a formulation of a mathematical theory of thermal viscoelasticity and the solutions of some boundary value problems.

Gurtin and Williams [35,36] have suggested that there are no *a priori* grounds to assume that the second law of thermodynamics for continuous bodies involve only a single temperature, i.e., it is more logical to assume a second law in which the entropy contribution due to heat conduction is governed by one temperature, that of the heat supply by another. Chen and Gurtin [37] and Chen et al. [38,39] have formulated a theory of heat conduction in deformable bodies which depends on two distinct temperatures – the conductive temperature ϕ and the thermodynamical temperature θ. The conductive temperature is due to the thermal processes and the heat exchange with the external world and the thermodynamical temperature is due to the mechanical processes inherent between the particles of elastic materials. Chen et al. [38] have pointed out that the difference between these two temperatures is proportional to heat supply and these temperatures become equal for time-independent situation in the absence of heat supply. However, for time-dependant cases the two temperatures are in general different, regardless of the heat supply. The key element that sets the two-temperature thermoelasticity apart from the classical theory of thermoelasticity is the material parameter a ($\geqslant 0$), called the temperature discrepancy. Specifically if $a = 0$, then $\phi = \theta$ and the field equations of two-temperature thermoelasticity reduce to those of classical theory of thermoelasticity. Youssef [40] extended this theory in the frame of generalized theory of heat conduction by introducing thermal relaxation parameters in the constitutive relations and proposed a two-temperature theory of generalized thermoelasticity. Uniqueness of the solution for this theory has also been derived by Youssef in the same article.