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Conjugate heat transfer in a plate – One surface at constant temperature and the other cooled by forced or natural convection

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ABSTRACT

Simple analytical solutions for the steady-state total heat transfer rate are presented for a flat plate cooled on one surface by forced or natural convection while the other side remains at uniform temperature. These solutions are also used to determine the unknown interface temperature of the surface. The solutions are based on one-dimensional conduction in the plate and on the heat transfer coefficients of an isothermal surface. The results are compared with those in the literature combined with a brief discussion of the most relevant studies in the field. In addition, presented are also numerically calculated results that take into account two-dimensional conduction in the plate and the effect of non-uniform surface temperature on convection. The calculations show that the results are valid also for thick plates. The solution procedure proved very accurate and produced new simple results for engineering applications.

1. Introduction

In conjugate heat transfer problems, a simultaneous solution is required for heat conduction in a solid and convection to an adjacent fluid because neither the temperature distribution nor the heat flux at the solid–fluid interface is known a priori, yet they contribute to solving the problem. Though the literature on conjugate heat transfer dates back to the 1960s', Dorfman's [1] recent book is the first comprehensive work on existing studies.

The most interesting feature in solving conjugate heat transfer problems in, for example, electronics cooling systems is usually the relationship between the total heat transfer rate and the maximum temperature. In general, if boundary conditions are specified for temperature, the total heat transfer rate is of interest. On the other hand, with a specified heat input, the total heat transfer rate is known and the maximum temperature should be determined.

Perhaps the best known conjugate heat transfer problem is that of Perelman [2] and Luikov [3], in which one side of the plate remains at a constant temperature while the other is heated or cooled by forced laminar convection (illustrated schematically in Fig. 1a for forced convection and in Fig. 1b for natural convection). As a result of non-uniform cooling, the temperature distribution on the plate surface remains unknown, as does the total heat transfer rate.

The distribution of the surface temperature in the plate in Fig. 1a with laminar forced convection has frequently been studied. For example, Payvar [4], Karvinen [5], and Mosaad [6] used integral boundary layer methods for convection and assumed a linear temperature profile across the plate thickness, ignoring stream-wise heat conduction. Trevino et al. [7] developed an analytical equation for total heat flux when the plate is close to isothermal. Because of their convection models, all these studies are, in fact, valid for Prandtl numbers larger than 0.5. Worth quoting are also the numerical results of Chida [8], which took into accounttwo-dimensional conduction in the plate. Most analytical results are valid only at large or small Brun numbers, whereas only few sophisticated solutions are valid everywhere [9].

Vynnycky and Kimura [10] made an important contribution to solving the problem in Fig. 1b by developing an approximate one-dimensional model, which does not solve the boundary layer equations at all but rather assumes the surface to be isothermal and uses the heat transfer coefficient of natural convection from an isothermal surface. Their results were experimentally verified by Kimura et al. [11] for three plates with different material properties using water for fluid. Vynnycky et al. [12] developed a corresponding one-dimensional model for the total heat transfer rate in forced convection, which introduced a connection between the mean surface temperature and the total heat transfer rate.

Natural convection has mostly been treated numerically by solving the boundary layer equations. Merkin and Pop [13] presented such a solution using an iterative finite-difference scheme whereas Yu and Lin solved the boundary layer equations numerically and presented a correlation for the surface temperature [14]. The effect of stream-wise conduction in the plate has been numerically studied by Miyamoto et al. [15], who also proposed a simple model for the surface temperature, based on the heat transfer coefficient of an isothermal surface.

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Nomenclature

Bi	mean Biot number of isothermal surface, ht/k_s
Bi_f	mean Biot number for forced convection, Eq. (6)
Bi_n	mean Biot number for natural convection, Eq. (10)
C	0.332 (lam. forced), 0.0287 (turb. forced)
c_p	fluid specific heat at constant pressure, J/kg K
c_T	Prandtl number function, Eq. (8)
F	non-dimensional heat transfer rate, Eq. (3)
F_f	solution of forced convection, Eq. (5)
F_n	solution of natural convection, Eq. (9)
g	gravitational acceleration, m/s^2
h	mean heat transfer coefficient of isothermal surface, W/m^2 K
k_f	fluid thermal conductivity, W/m K
k_s	solid thermal conductivity, W/m K
L	plate length, m
m	1/2 (lam. forced), 4/5 (turb. forced), 3/4 (lam. natural)
n	1/3 (lam. forced), 3/5 (turb. forced)
Nu	mean Nusselt number of isothermal surface, hx/k_f
Pr	Prandtl number, $\mu c_p/k_f$
$q(x)$	local surface heat flux, W/m^2
Re	Reynolds number, $\rho Ux/\mu$,
s	dummy integration variable, m
t	plate thickness, m
T_B	constant surface temperature, K
$T_s(x, y)$	plate temperature, K
$T_w(x)$	local surface temperature, K
$\bar{T}_w$	mean surface temperature, K
T_∞	ambient temperature, K
U	ambient flow velocity, m/s
x	coordinate in flow direction, m
y	coordinate normal to plate surface, m

Greek symbols

β	1/3 (lam. forced), 1/9 (turb. forced)
β_T	fluid volumetric thermal expansion coefficient, K^{-1}
γ	3/4 (lam. forced), 9/10 (turb. forced)
$\theta(x)$	non-dimensional surface temperature, Eq. (14)
$\theta_f(x)$	solution for forced convection, Eq. (16)
$\theta_n(x)$	solution for natural convection, Eq. (21)
μ	fluid viscosity, kg/m s
ρ	fluid density, kg/m^3
Φ	total heat transfer rate (per unit depth) up to x, $\int_0^x q(x)dx$, W/m

Though the literature contains many solutions, the problem has nevertheless not been fully solved. Most of the surface temperature solutions are limited to laminar forced or natural convection while no solutions exist for turbulent forced convection. Most of the results are difficult to apply.

In this paper, one-dimensional models are extended to include both natural and forced convection. General simple analytical solutions are presented for the total heat transfer rate and for the temperature distribution of the convectively cooled surface. The surface temperature of the plate is derived from the total heat transfer rate with an as yet unknown method. The accuracy of the very simple results was verified by comparing them with those in the literature and with our numerical results, which took into account two-dimensional conduction in the plate and the effect of non-uniform surface temperature on convection.

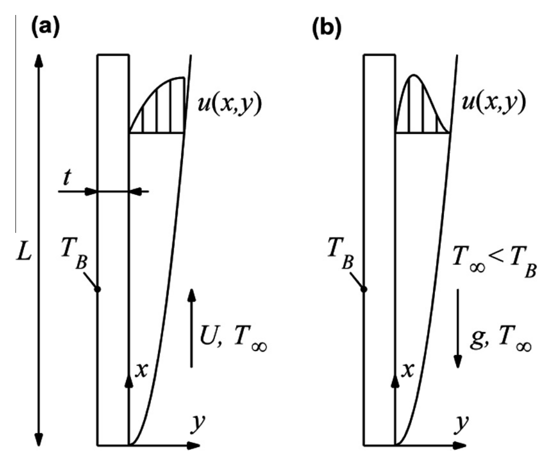

Fig. 1. Schematics of the problem of forced (a) and natural convection (b). Constant temperature T_B at one side and other cooled by convection.

2. Total heat transfer

The total heat transfer from the plate to its surroundings in Fig. 1 is unknown. In a study of total heat transfer from plate fins, excellent results were obtained by using the mean heat transfer coefficients of an isothermal surface in a fin theory [16]. A similar approach was applied to the classical problem in Fig. 1. If the surface temperature is assumed constant and equal to $\bar{T}_w$, the total heat transfer rate from the leading edge of the plate up to point x can be expressed either by calculating conduction through the wall or convection from the surface to the surroundings as follows:

$$\Phi = xk_s\frac{T_B - \bar{T}_w}{t} = hx(\bar{T}_w - T_\infty), \tag{1}$$

where h is the average heat transfer coefficient of the surface at a constant temperature $\bar{T}_w$.

If $\bar{T}_w$ is eliminated from Eq. (1), the total heat flux can be expressed as

$$\Phi = \frac{T_B - T_\infty}{\frac{1}{hx} + \frac{t}{k_s x}}. \tag{2}$$

Eq. (2) is valid for both natural and forced convection when the mean heat transfer coefficients are inserted in it. Very general results are obtained using the non-dimensional heat flux and non-dimensional mean surface temperature, which are obtained from Eqs. (1) and (2) as

$$F = \frac{\Phi t}{k_s x(T_B - T_\infty)} = \frac{1}{1 + \frac{k_s}{th}} = \frac{T_B - \bar{T}_w}{T_B - T_\infty}, \tag{3}$$

In Eq. (3), $\bar{T}_w$ is actually the mean surface temperature, which is verified on the basis of numerical results below.

2.1. Forced convection

Convection heat transfer from isothermal surfaces is usually expressed using the mean Nusselt numbers $\mathrm{Nu} = hx/k_f$, found in the literature. The mean Nusselt numbers for forced convection ($\mathrm{Pr} \geqslant 0.5$) can be expressed as

$$\mathrm{Nu} = \frac{C}{m}\mathrm{Re}^m\mathrm{Pr}^n, \tag{4}$$

where $C = 0.332$, $m = 1/2$ and $n = 1/3$ for a laminar boundary layer and $C = 0.0287$, $m = 4/5$, $n = 3/5$ for a turbulent boundary layer, respectively [18].