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Triple diffusive free convection along a horizontal plate in porous media saturated by a nanofluid with convective boundary condition

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ABSTRACT

The steady triple diffusive boundary layer free convection flow past a horizontal flat plate embedded in a porous medium filled by a water-based nanofluid and two salts is investigated numerically. The plate is assumed to be convectively cooled by a surrounding fluid. It is assumed that there is no nanoparticle flux at the surface and the effect of thermophoresis is taken into account in the boundary condition. The effects of Brownian motion and thermophoresis parameters are also introduced through Buongiorno model in the governing equations. Using appropriate similarity variables, the partial differential equations are transformed to ordinary (similarity) equations, which are then solved numerically using an implicit finite difference method with quasi-linearization technique. The effects of the buoyancy ratio, regular Lewis numbers and modified Dufour parameters of both salts and nanofluid parameters on the flow and heat transfer are investigated. It is found that the heat transfer rate increases as we include nanoparticles and salts.

1. Introduction

Conventional heat transfer fluids such as water, ethylene glycol mixture and engine oil have limited heat transfer capabilities due to their low thermal conductivity in enhancing the performance and compactness of many engineering electronic devices. In contrast, metals have thermal conductivities up to three times higher than these fluids. Thus it is naturally desirable to combine the two substances to produce a medium for heat transfer that would behave like a fluid, but has the thermal conductivity of a metal. So there is a strong need to develop advanced heat transfer fluids with substantially higher conductivities to enhance thermal characteristics. Nanofluids may be used in various applications which include electronic cooling, vehicle cooling transformer, coolant for nuclear reactors [10]. Modern nanotechnology can produce metallic or nonmetallic particles with average sizes below 100 nm in traditional heat transfer fluids such as: water, oil, and ethylene glycol. Nanofluids (nanoparticle fluid suspension) is the term introduced by Choi [7] to describe this new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their host (based) fluids. Nanofluids technology, a new interdisciplinary field of great importance, where nanoscience, nanotechnology, and thermal engineering meet, has largely developed over the past two decades. However, the study of nanofluids is still at its early stage, and it seems very difficult to have a precise idea on the way the use of nanoparticles acts in fluid flow and heat transfer, and complementary works are needed to understand the heat transfer characteristics of nanofluids and identify new and unique applications for these fluids. As it is already well-known, the main scope of the nanofluids is the enhancement of thermal conductivity of liquids, which is an extremely important topic from the energy efficiency point of view. The latest technique on this challenging subject is the using of addition of some particle into the base fluid. In this context, the theory of nanofluids became more attractive in recent years due to easy production methods and inexpensive price. Also, thermal conductivity of nanofluids relative to the base fluids is very high. Thus, nanofluids can be applied in many energetical systems such as cooling of nuclear systems, radiators, natural convection in enclosures, etc. Choi et al. [8] showed that the addition of small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids which enhanced the thermal conductivity of the fluid up to approximately two times. Experimental studies [9,19] show that even with small volumetric fraction of nanoparticles (usually less than 5%) the thermal conductivity of the base fluid is enhanced by 10–50% with a remarkable improvement in the convective heat transfer coefficient. The characteristic feature of nanofluid is the thermal conductivity enhancement, a phenomenon observed by Masuda et al. [18]. This suggested the possible use of nanofluids in advanced nuclear systems [4]. Khanafer et al. [15],

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Nomenclature

A	actual value of each graph
β_{C_1}, β_{C_2}	volumetric solutal expansion coefficients of the fluid
C	nanoparticle volume fraction
C_1, C_2	solutal concentration of salts 1 and 2
C_{1w}, C_{2w}	solutal concentration of salts 1 and 2 at the wall
C_∞	ambient nanoparticle volume fraction
$C_{1\infty}, C_{2\infty}$	ambient solutal concentration of salts 1 and 2 attained as y tends to infinity
D	default value
D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
D_{CT}	Soret diffusivity
D_{TC}	Dufour diffusivity
D_{Sm}	solutal diffusivity of porous medium
g	acceleration due to gravity
K	permeability of the porous medium
Ld_1, Ld_2	Dufour-solutal Lewis numbers of salts 1 and 2
Ln	nanofluid Lewis number
Nb	Brownian motion parameter
Nc	regular double-diffusive buoyancy parameter
Nc_1, Nc_2	buoyancy ratios of salts 1 and 2
Nr	nanofluid buoyancy ratio
Nd_1, Nd_2	modified Dufour parameters of salts 1 and 2
Le_1, Le_2	regular Lewis numbers of salts 1 and 2
Nu_x	local Nusselt number
Nur	reduced Nusselt number
Ra_x	local Rayleigh number
Sh_x^1	local solutal Sherwood number for salt 1
Sh_x^2	local solutal Sherwood number for salt 2
Shr^1	reduced solutal Sherwood number for salt 1
Shr^2	reduced solutal Sherwood number for salt 2
T	local fluid temperature
T_∞	ambient temperature
u,v	velocity components along x and y directions
x	coordinate along the plate
y	coordinate normal to the plate.

Greek symbols

α_m	thermal diffusivity of porous medium
β_T	volumetric thermal expansion coefficient of the fluid
χ_1, χ_2	dimensionless solutal concentration of salt 1 and salt 2
ϕ	rescaled nanoparticle volume fraction
η	similarity variable
μ	absolute viscosity of the base fluid
ν	kinematic viscosity of the fluid
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_m$	effective heat capacity of the porous medium
$(\rho c)_p$	effective heat capacity of the nanoparticle material
τ	ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid
θ	dimensionless temperature
ψ	stream function

and Tiwari and Das [24] numerically investigated buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. Another mathematical model of nanofluids is that proposed by Buongiorno [5]. It seems, however, that this model is rather limited because it includes different slip mechanisms between nanoparticles and base fluid. Thus, Buongiorno [5] has indicated that there are many slip mechanisms such as inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity, while the mathematical model proposed by Khanafer et al. [15], and Tiwari and Das [24] contains only the main parameter: volume fraction parameter of the nanofluids.

In a series of pioneering papers, Nield and Kuznetsov [20,21], and Kuznetsov and Nield [16,17] have used the mathematical nanofluid model proposed by Buongiorno [5] to study some problems on viscous (regular) fluids and porous media filled by nanofluids. Several authors have tried to establish convective transport models for nanofluids. Khan and Pop [13] analyzed the development of the steady boundary layer flow, heat transfer and nanoparticle fraction over a stretching surface in a nanofluid. Bachok et al. [2] investigated boundary-layer flow of nanofluids over a moving surface in a flowing fluid. Uddin et al. [25], Khan and Pop [14] and Aziz et al. [1] studied numerically the free convective flow of a nanofluid over a horizontal plate. They employed the mathematical nanofluid model proposed by Buongiorno [5] and solved the problems using an implicit finite difference numerical method. They assumed that the plate is embedded in a porous medium filled by a water-based nanofluid with or without gyrotactic microorganisms.

In a very recently paper, Rionero [23] has shown that the presence of more than one chemical dissolved in fluid mixtures is very often requested for describing natural phenomena (contaminant transport, underground water flow, acid rain effects, warning of the stratosphere). Thus, these authors have studied a triple convective–diffusive fluid mixture on a horizontal plate heated from below and salted from above and below. It is important to mention here that the conditions sufficient for (i) inhibiting the onset of convection and (ii) guaranteeing the global nonlinear stability of the thermal conduction solution and the absence of subcritical instabilities were obtained. Using the idea of Rionero [23], we consider the triple convective–diffusive fluid mixture ($m = 2$) along a horizontal flat plate embedded in a porous medium saturated by a nanofluid with convective boundary condition. As far as we know, the results of this paper are completely new in the existing literature and, in view of their simplicity, useful in different applications. In fact the paper extends the classical work of Cheng and Chang [6], Khan and Aziz [12], and Kuznetsov and Nield [17] to the case of a nanofluid and two salts.

It should be, however, mentioned that Nield and Kuznetsov [20], Nield and Kuznetsov [21] have assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix. On the other hand, it is very important to explain how nanofluid flow is possible in a porous medium. Without special precautions, nanoparticles will be simply absorbed by the porous matrix. Basically, the porous matrix will work as a filter for nanoparticles. This situation has been very well described, explained, and modeled in the recent papers by Wu et al. [26,27]. The physical situation described in these papers show that the work on porous media filled by nanofluids are not just a mathematical exercise, but are based on deep physical understanding of nanofluid flows. This demonstrates that we are simulating here a real physics problem of free convection flow and heat transfer past a horizontal flat plate in a porous medium filled by a nanofluid.

2. Basic equations

Steady boundary layer free convection of a nanofluid past the upward face of a horizontal flat plate embedded in a porous