International Journal of Heat and Mass Transfer 66 (2013) 699–705

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Analytical solution of oscillating flow in a capillary tube

D. Yin [a], H.B. Ma [a,b,*]

[a] Institute of Marine Engineering and Thermal Science, College of Marine Engineering, Dalian Maritime University, Dalian 116026, China
[b] Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

ARTICLE INFO

Article history:
Received 25 January 2013
Received in revised form 23 July 2013
Accepted 23 July 2013
Available online 23 August 2013

Keywords:
Oscillating flow
Nusselt number

ABSTRACT

This paper presents an analytical study of oscillating flow in a capillary tube to determine the oscillatory flow effect on the heat transfer coefficient and find whether there exists an optimum condition for the maximum heat transfer coefficient. Based on a uniform heat flux boundary condition, analytical solutions of temperature distribution and Nusselt numbers are obtained for an oscillating laminar flow, which can be used to analyze the effects of flow type including thermal properties on the heat transfer performance. The result shows that the dimensionless oscillating frequency, ω^*, amplitude, γ, and Prandtl number, Pr, are primary factors affecting the heat transfer performance of an oscillating flow in a capillary tube, which can be used to determine an optimum condition of the maximum heat transfer coefficient.

1. Introduction

In 1929, Richardson and Tyler [1] found that when a pulsating flow existed in a circular tube, the velocity profile near the wall surface is different, which directly affects wall shear stress and heat transfer. Due to unique features and applications, extensive investigations have been conducted on oscillating flow and its effect on heat transfer over the last several decades [3–12]. Womersley [2] studied the phase-lag phenomenon between the oscillating pressure and flow rate, and presented an analytical solution of flow rate in a small round tube at a condition of oscillating pressure gradient. His calculation showed that a phase-lag between pressure gradient and flow rate existed, which is analogous with the phase-lag between voltage and current in a conductor carrying alternating current. Womersley's investigation provided a new way to analyze the oscillating flow in a capillary pipe.

In the 1990s, Zhao and Cheng [3] developed a numerical solution to determine laminar forced convection in a heated pipe subjected to a reciprocating flow. The results showed that the averaged heat transfer rate is found to increase with both the kinetic Reynolds number and the dimensionless oscillation amplitude but decreases with the ratio of length to diameter. Subsequently, they carried out an experimental study [4] and found that the space-cycle averaged Nusselt number predicted by the numerical solution was in good agreement with their experimental data. Moschandreou and Zamir [5] analyzed oscillating flow in a tube with constant heat flux. The results indicated that there exists a frequency which results in an increase of a fluid's bulk temperature and an increase in the Nusselt number, where the Prandtl number plays a key role.

Pendyala et al. [6] experimentally studied the oscillation effect on heat transfer in a vertical tube with a constant amplitude of 0.125 m and frequencies ranging from 0.133 to 0.5 Hz. The experimental results show that with this low frequency, the oscillating motion can enhance heat transfer. Because the total length of Pendyala's test section was 1.01 m with an inner diameter of 0.016 m, the entrance influence played an important role that was different from the numerical prediction of a fully developed laminar flow. The entrance effect developed nonlinear hydraulic and thermal boundary conditions in an oscillating flow. Sert and Beskok [7] studied laminar, forced convection heat transfer for reciprocating, two-dimensional channel flows by numerical simulations. Uniform heat flux and constant temperature boundary conditions were imposed on certain regions of the top surface, while the bottom surface was kept insulated. The results showed that steady unidirectional forced convection is more effective than reciprocating forced convection. Yu et al. [8] theoretically studied pulsating laminar heat convection in a circular tube with constant heat flux. The results indicated that pulsation has no effect on the time-averaged Nusselt numbers for pulsating convection heat transfer, which was based on an amplitude $\gamma = 0.01$, $Pr = 0.1$, and dimensionless frequency $\omega^* = 0.1$. Guo and Sung [9] studied many versions of the Nusselt number to clarify conflicting results in heat transfer characteristics for pulsating flow in a pipe. They concluded that a new definition of the Nusselt

* Corresponding author at: Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA. Tel.: +1 573 884 5944.
E-mail address: mah@missouri.edu (H.B. Ma).

0017-9310/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijheatmasstransfer.2013.07.073

Nomenclature

a	constant defined by Eq. (22)
b	constant defined by Eq. (23)
c	constant defined by Eq. (33)
A	constant defined by Eq. (58)
B	constant defined by Eq. (59)
C	constant defined by Eq. (41)
E	constant defined by Eq. (54)
f	function defined by Eq. (32)
F	function defined by Eq. (19)
g	function defined by Eq. (32)
J	Bessel function
k	thermal conductivity, W/mK
Nu	Nusselt number
p	pressure, N/m^2
Pr	Prandtl number
q	heat flux, W/m^2
r	radius, m
Re	Reynolds number
s	solution of Bessel's transcendental equations
t	time, s
T	temperature, K
u	velocity, m/s
x	axial position, m
X	dimensionless axial position

Greek

α	thermal diffusivity, m^2/ s
γ	dimensionless amplitude of the pressure
μ	dynamic viscosity, N s/m^2
v	kinematic viscosity, N s-m/kg
ρ	density, kg/m^3
ω	angular velocity, rad/s
Θ	dimensionless temperature

Subscripts

0	bottom
bt	instantaneous bulk temperature
m	mean value
s	steady state
t	transient state
w	wall
Δ	temporary variable

number was needed to scrutinize the effects of the oscillation amplitude and frequency on heat transfer. It was found that, for small amplitude, both heat transfer enhancement and reduction were detected depending on the oscillation frequency, but for large amplitude, heat transfer due to oscillation is always augmented. In order to further eliminate existing confusion at various levels, the pulsation effect on heat transfer is theoretically investigated by Hemida et al. [10]. Different types of thermal boundary conditions were considered including the thermally developed/developing regions. They concluded that when linear boundary conditions exist, the time average heat transfer coefficient tends to be constant or negative with very small differences, but when non-linear boundary conditions exist, pulsation or oscillation may result in a noticeable enhancement of the time-averaged Nusselt number. Guo et al. [11] investigated pulsating flow in a circular pipe with partial filling of porous media. It was found that a porous layer added to the inner surface of the pipe can help to enhance heat transfer of oscillating flow depending on the Darcy number, oscillating frequency, and oscillating amplitude. Habib et al. [12] experimentally studied the effects of the Reynolds number and pulsation frequency on heat transfer of laminar oscillating pipe flow and found that the oscillating frequency significantly affected the relative mean Nusselt number, while the Reynolds number had almost no effect on the Nusselt number. More importantly, a significant improvement in the heat transfer coefficient was found in the entrance region of an oscillating flow. From previous investigations [1–12], it was determined that when a nonlinear boundary condition exists, an oscillating motion can enhance heat transfer. Nonlinear boundary conditions can result from the developing entrance region, turbulent flow, nonlinear porous medium, or a radiation condition.

In this paper, an oscillating flow in a capillary round tube is studied to determine the oscillating pressure effect on fluid flow and heat transfer. Based on a uniform heat flux boundary condition, analytical solutions of temperature distribution and the Nusselt number, results are obtained for calculating an oscillating flow. The focus will be on the effects of thermal and mechanical properties on the heat transfer performance of an oscillating flow in order to determine the primary factors optimizing heat transfer in a capillary round tube.

2. Theoretical modeling

Fig. 1 shows a schematic of an oscillating Newtonian flow in a capillary round tube with a radius of r_0. For the oscillating flow shown in Fig. 1, it is assumed that (1) the flow is laminar and fully developed; (2) the flow is incompressible; (3) the surface tension effect is not considered; (4) a uniform heat flux is added on the boundary; (5) the driving force added to the fluid flow is a sinusoidal pressure gradient, i.e.,

$$\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s (1 + \gamma \cos(\omega t)) \tag{1}$$

where γ is a constant that controls the amplitude of the pressure wave form. The equations governing fluid flow and heat transfer can be written as:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r\partial r}\right) \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} = \alpha\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

The boundary conditions for governing Eqs. (2) and (3) can be found as:

$$\begin{aligned} &\frac{\partial u}{\partial r} = 0 \text{ and } \frac{\partial T}{\partial r} = 0 \text{ at } r = 0 \\ &k\frac{\partial T}{\partial r} = q_w \text{ at } r = r_0 \end{aligned} \tag{4}$$

respectively. Considering:

$$r^* = \frac{r}{r_0}, \quad \omega^* = \frac{\omega r_0^2}{v}, \quad t^* = \frac{vt}{r_0^2}, \quad u^* = \frac{u}{u_m}, \quad \Theta = \frac{T - T_0}{q_w r_0/k},$$

$$X = \frac{4x}{Re_m \Pr r_0}, \text{ and } Re_m = \frac{2u_m r_0}{v}.$$

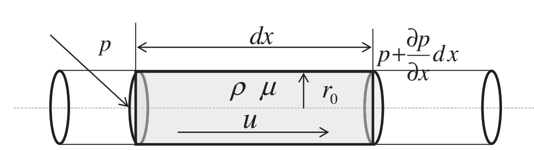

Fig. 1. The physical model.