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Technical Note

Least-squares natural element method for radiative heat transfer in graded index medium with semitransparent surfaces

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ABSTRACT

The least-squares natural element method (LSNEM) is employed for solving radiative heat transfer problem in two-dimensional semitransparent graded index medium with semitransparent and diffusely reflecting surfaces. LSNEM is an extension of natural element method (NEM). Unlike the NEM based on Galerkin discretization, the least-squares weighted residuals approach is employed to spatially discretize the radiative heat transfer equations in LSNEM. Radiative heat transfer problem in rectangular enclosure filled with graded index medium having opaque surfaces is examined to verify the LSNEM. Afterwards, radiative heat transfer in the semicircular enclosure with an inner circle filled with graded index medium is studied. Two kinds of refractive index distributions are considered. The studies show that the refractive index distributions and optical boundary conditions of the surfaces greatly influence the radiative transfer. Effects of the extinction coefficient are investigated on the temperature distributions and radiative heat fluxes.

1. Introduction

The media with a spatial variable refractive index distribution (graded index, GRIN) exist broadly in nature. Typical examples include the earth's atmosphere [1], the stellar atmosphere [2], jellyfish eyes [3] and human eyes [4]. Artificial GRIN materials have a variety of promising applications [4]. Researchers produced various types of GRIN materials to improve the quality of optical systems. Like conventional optical materials, heat-transfer processes must be considered in the manufacture and application of the GRIN materials.

Radiative heat transfer in semitransparent GRIN medium is of great importance in thermo-optical systems, and has evoked the wide interest of many researchers. Because a ray goes along a curved path determined by the Fermat principle, the solution of radiative transfer in a GRIN medium is more difficult than that in a uniform refractive index medium.

Since 2000, researchers have been working on thermal radiative transfer within GRIN media. Ray tracing and Monte-Carlo techniques are particularly well adapted to solving radiative transfer in the medium with specular boundaries and widely used to solve radiative transfer in the graded index medium. Ben Abdallah and coworkers [5–8] developed a curved ray-tracing technique to analyze radiative heat transfer in an absorbing–emitting semitransparent medium with variable spatial refractive index. Based on the works of Ben Abdallah and coworkers, Huang et al. [9,10] and Xia et al. [11] presented a combined curved ray-tracing and pseudo-source adding method for radiative heat transfer in one-dimensional semitransparent medium with graded refractive index.

Because of a large number of rays to be launched, the ray tracing method is time consuming and difficult to settle the problem of radiative transfer in multidimensional complex geometries. In order to overcome these disadvantages, other approaches different from the ray-tracing based methods which are based on the discretization of radiative transfer equation were proposed. Lemonnier et al. [12] proposed a discrete ordinates method (DOM) which could be used in the 1D slab. Afterwards, Liu derived another kind of DOM for graded index radiative transfer, expressed in a 3D Cartesian coordinates system [13] and a cylindrical one [14] respectively. Based on the DOM proposed by Liu, different numerical techniques have been developed for solving the radiative transfer equation (RTE) in multi-dimensional graded index media, including the finite volume method [13], proposed a modified FVM to solve the problem of radiative transfer in a graded index media. Most recently, Zhang et al. [15] extended the hybrid finite volume with finite element method (hybrid FVM/FEM) to solve the radiative transfer in a graded index medium. Using a diffuse approximation meshless (DAM) method, Wang [16] studied the radiative transfer in a graded index medium.

Recently, some researchers also introduced meshless methods to solve thermal radiative transfer within GRIN media [16]. The

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meshless methods are alternative approaches to the mesh based numerical methods to solve partial differential equations (PDE). A common feature of all the meshless methods is that the approximation of field variables is constructed entirely based on a group of discrete nodes and no predefined nodal connectivity is required. The natural element method (NEM) proposed by Braun and Sambridge [17] and Sukumar et al. [18] is a relatively new meshless Galerkin procedure based on the natural neighbor interpolation scheme, which in turn relies on the concepts of Voronoi diagrams and Delaunay triangulation to build Galerkin trial and test functions. Compared to the MLS approximation, some of the most important advantages of natural neighbor interpolants are the properties of interpolation of nodal data, easiness of imposing essential boundary conditions, and a well-defined and robust approximation with no user-defined parameter on non-uniform grids.

Most recently, we has been introduced the NEM to solve radiative heat transfer problems [19,20]. In our work [19], several benchmark problems of two dimensions were solved using NEM, and the results shows that the NEM for radiative heat transfer is efficient, accurate and stable. Afterwards, we [20] extended the natural element solutions to deal with the coupled heat transfer problem in an irregular enclosure under mixed boundary conditions.

The research works mentioned above are all dealing with radiative heat transfer in semitransparent medium with opaque boundaries. To the knowledge of the authors, there are no research works have been carried out to analyze the radiative heat transfer in multidimensional graded index media having semitransparent surfaces. For semitransparent surfaces, radiative energy will be reflected and transmitted when it reaches the semitransparent surfaces, and total reflection will occur therein when the incident angle is greater than the critical angle. Therefore, the problem caused by semitransparent surfaces becomes more complex. A few papers discussed the problems in uniform index medium with semitransparent boundary conditions [21,22]. In the treatment of boundary conditions of semitransparent surfaces, complex phenomena such as refraction through a semitransparent limit and (or) convective heat exchange with the surroundings were taken into account.

In this paper, we aims at further extending the application of NEM to solve radiative transfer problems in two-dimensional GRIN media having semitransparent and diffusely reflecting surfaces. The least-squares weighted residuals approach is employed to spatially discretize the radiative heat transfer equations. To distinguish with the traditional NEM that is called as a meshless Galerkin procedure, we name the method least-squares natural element method (LSNEM). We validate the correctness of the solution of the LSNEM to radiative transfer in semitransparent medium with graded refractive index by comparing with the results obtained from the literature. The radiative heat transfer problems involving semitransparent surfaces are investigated. For the semicircular medium with an inner circle, we consider two kinds of refractive index distributions in the medium. Effects of various parameters such as the extinction coefficient and the scattering albedo are examined on the temperature distributions.

2. Mathematical formulation

We consider radiative heat transfer problems in enclosures filled with absorbing, emitting and scattering medium. The surfaces bounding the physical domain are assumed gray and diffusely reflecting semitransparent or opaque boundaries.

The discrete ordinate equation of radiative transfer in a multidimensional graded index medium can be written as [13]:

$$\begin{aligned}\mathbf{s}^m\cdot\nabla I^m(\mathbf{r},\mathbf{s}) &+ \frac{1}{\sin\theta^m}\frac{\partial}{\partial\theta}\left\{\left[I^m(\mathbf{r},\mathbf{s})(\xi\boldsymbol{\Omega}-\mathbf{k})\cdot\frac{\nabla n}{n}\right]\right\}\\ &+\frac{1}{\sin\theta^m}\frac{\partial}{\partial\varphi}\left\{I^m(\mathbf{r},\mathbf{s})\left[\mathbf{s}_1\cdot\frac{\nabla n}{n}\right]\right\}\\ &+(\kappa_a+\kappa_s)I^m(\mathbf{r},\mathbf{s}) = n^2\kappa_a I_b\\ &+\frac{\kappa_s}{4\pi}\sum_{m'=1}^{M} I^{m'}\Phi^{m',m}w^{m'}\end{aligned}\tag{1}$$

where $\mathbf{s} = \mathbf{i}\mu + \mathbf{j}\eta + \mathbf{k}\xi = \mathbf{i}\sin\theta\cos\varphi + \mathbf{j}\sin\theta\sin\varphi + \mathbf{k}\cos\theta$. The radiative boundary condition for a reflecting semitransparent surface can be expressed as:

$$I_w^m = (1-\rho_O)I_0 + \frac{\rho_I}{\pi}\sum_{n_w\cdot s^{m'}>0}\left|\mathbf{n}_w\cdot\mathbf{s}^{m'}\right|I_w^m w^{m'},\quad \mathbf{n}_w\cdot\mathbf{s}^m<0 \tag{2}$$

where ρ_O is the external diffuse reflectivity, ρ_I is the internal diffuse reflectivity, T_e is the temperature of the environment.

The external diffuse reflectivity ρ_O can be expressed as [23]:

$$\begin{aligned}\rho_o(n') = \frac{1}{2} &+ \frac{(3n'+1)(n'-1)}{6(n'+1)^2} + \frac{n'^2(n'^2-1)^2}{(n'^2+1)^3}\ln\frac{(n'-1)}{(n'+1)}\\ &- \frac{2n'^3(n'^2+2n'-1)}{(n'^2+1)(n'^4-1)} + \frac{8n'^4(n'^4+1)}{(n'^2+1)(n'^4-1)^2}\ln(n')\end{aligned}\tag{3}$$

where $n' = n/1 = n$.

Considering the effect of total reflection, the internal diffuse reflectivity of is [23]

$$\rho_I(n') = 1 - \frac{1}{n'^2}[1-\rho_o(n')] \tag{4}$$

Using the piecewise constant angular (PCA) quadrature and the step scheme for the treatment of the angular redistribution terms in Eq. (1)

$$\mathbf{s}^{m,n}\cdot\nabla I^{m,n} + \tilde{\beta}^{m,n}I^{m,n} = S^{m,n} \tag{5}$$

where the effective extinction coefficient $\tilde{\beta}(\mathbf{r})$ and effective source term $S^{m,n}(\mathbf{r})$ are defined as

$$\begin{aligned}\tilde{\beta}^{m,n}(\mathbf{r}) = &\frac{1}{w_\theta^m}\max\left(\chi_\theta^{m+1/2,n},0\right) + \frac{1}{w_\theta^m}\max\left(-\chi_\theta^{m-1/2,n},0\right)\\ &+\frac{1}{w_\varphi^n}\max\left(\chi_\varphi^{m,n+1/2},0\right) + \frac{1}{w_\varphi^n}\max\left(-\chi_\varphi^{m,n-1/2},0\right)\\ &+(\kappa_a+\kappa_s)\end{aligned}\tag{6a}$$

$$\begin{aligned}S^{m,n}(\mathbf{r}) = n^2\kappa_a I_b &+ \frac{\kappa_s}{4\pi}\sum_{m'=1}^{N_\theta}\sum_{n'=1}^{N_\varphi} I^{m',n'}\Phi^{m',n';m,n}w_\theta^{m'}w_\varphi^{n'}\\ &+\frac{1}{w_\theta^m}\max\left(-\chi_\theta^{m+1/2,n},0\right)I^{m+1,n}\\ &+\frac{1}{w_\theta^m}\max\left(\chi_\theta^{m-1/2,n},0\right)I^{m-1,n}\\ &+\frac{1}{w_\varphi^n}\max\left(-\chi_\varphi^{m,n+1/2},0\right)I^{m,n+1}\\ &+\frac{1}{w_\varphi^n}\max\left(\chi_\varphi^{m,n-1/2},0\right)I^{m,n-1}\end{aligned}\tag{6b}$$

The recursion formulas for $\chi_\theta^{m-1/2,n}$ and $\chi_\varphi^{m-1/2}$ are giving as following:

$$\begin{aligned}&\chi_\theta^{m+1/2,n} - \chi_\theta^{m-1/2,n} = \frac{w_\theta^m}{\sin\theta^m}\left[\frac{\partial(\xi\boldsymbol{\Omega})}{\partial\theta}\cdot\frac{\nabla n}{n}\right]_{\Omega=\Omega^{m,n}}\\ &\chi_\theta^{1/2,n} = \chi_\theta^{N_\theta+1/2,n} = 0\end{aligned}\tag{7}$$