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Effect of mutual location and the shape of heaters on the stability of thin films flowing over locally heated surfaces

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ABSTRACT

We consider the motion of a liquid film falling down a locally heated planar substrate. Marangoni effect due to the local temperature gradients at the free surface induces a three dimensional interfacial deformation near the vicinity of the heater. The objective of the present paper is to study the influence of streamwise arrangement of the heaters of rectangular shape or with infinite spanwise lengths on the structure of interfacial deformation and the stability of the film. The problem is studied in the framework of longwave theory. We studied the influence of mutual location of heaters on the steady state of the film. As the film is heated by heaters with infinite/finite spanwise lengths, 2D/3D steady states exist. A linear stability analysis is performed with respected to both the 2D and 3D basic state. The results show that the mutual location of heater plays an important role in the stability and transient behavior of the film for 2D steady state. However, the 3D steady state of a film heated by rectangular heaters is linear stable. This means that the 3D steady state is a coherent structure in films heated by rectangular heaters.

1. Introduction

The dynamics, stability and rupture of thin liquid films are encountered in many areas of engineering, geophysics, and biophysics, including cooling technologies, nanofluidics, microfluidics, and coating. The effects of thermocapillarity on the instability of flow in thin liquid layers on a solid support have been extensively studied both theoretically and experimentally. For thin films uniformly heated from below, surface-tension-driven Bénard convection can exhibit a longwave primary instability [1] that differs from the classical Marangoni instability with a short wavelength reported by Pearson [2].

In recent years, increasing performance demands in semiconductor technology, including shrinking feature size, increasing transistor density, and faster circuit speeds, have resulted in very high chip power dissipation and heat fluxes. It is also leading to greater non-uniformity of on-chip power dissipation, creating localized, sub-millimeter hot spots, often exceeding 1 kW/cm^2 in heat flux, which can degrade the processor performance and reliability [3]. Similar developments are underway in microwave integrated circuits and power amplifier chips, with even higher localized heat fluxes and heat densities. The industrial and technological applications mentioned above involve thin liquid films on locally heated substrates. To avoid the reduction of their performance by film breakdown it is of crucial importance to understand when and why instabilities arise that may result in rupture of the film. Understanding the physical mechanisms of instability and rupture behavior is also highly desirable for the requirement of seeking effective ways to suppress the rupture of locally heated films.

Kabov [4] first studied the motion of the flow in a locally heated film experimentally. Subsequent experimental studies on the instability of locally heated falling films have been reported in Refs. [5–7]. These experimental results reveal the occurrence of novel effects: the competing flows produce a horizontal band of increased film thickness at the upper edge of the heater ('horizontal bump'), which may become unstable and develop rivulets periodic in the direction transverse to the flow. For more details of the phenomenon of 'horizontal bump' and 'rivulet structures', we refer the reader to the review article by Kabov [8].

Skotheim et al. [9] applied longwave theory to study the instability with respect to 2D stead basic state of a falling film over a locally heated substrate. The authors derived a Benney like equation to describe the nonlinear evolution of the film, in which the inertial terms are neglected. Their results show that a rivulet instability occurs with a finite transverse wavelength. Moreover, the calculated film

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Nomenclature

Bi	Biot number
Bo	Bond number
d	distance between two heaters
$\mathbf{g}$	vector of gravitational acceleration
g	gravitational acceleration
h	film thickness
H	far upstream film thickness
$\hat{H}$	amplitude of the normal mode disturbance of the film thickness
k	wavenumber
$\mathbf{L}$	linear operator of the stability problem
L	length scale in the streamwise direction
L_x, L_y	wavelengths in the x and y directions
l_x, l_y	sizes of the heater in the x and y directions
Ma	Marangoni number
$\mathbf{n}$	normal vector to the surface
p	pressure
$\mathbf{q}$	local flow rate
r	ratio of l_y/l_x
T	temperature
ΔT	temperature difference in the film
$\mathbf{t}$	tangential vector on the surface
t	time
$\mathbf{u}$	velocity vector
u	velocity components in the x direction
v	velocity components in the y direction
w	velocity components in the z direction
(x,y,z)	Cartesian coordinates

Greek symbols

α	Newtonian's heat transfer coefficient
$\mathbf{\Gamma}$	stress tensor
γ	surface tension variation with temperature
ϵ	small parameter for longwave expansion
θ	incline angle of the substrate
ρ	density
μ	dynamic viscosity
ν	kinematic viscosity
κ	thermal diffusivity
χ	thermal conductivity
σ	surface tension
ω	frequency of the disturbance

Subscripts

0	base state
r, i	real,imaginary part
∞	ambient gas phase far away from the interface

profile and wavelength of the most unstable mode at the instability threshold are in qualitative agreement with the experimental results. Kalliadasis et al. [10] studied the same problem as that in Ref. [9] using an integral-boundary-layer approximation of the Navier–Stokes equations. The computations have demonstrated that the free surface develops a bump in the region where the wall temperature gradient is positive. The results of the linear stability of this bump with respect to disturbances in the spanwise direction revealed the existence of both a discrete and an essential spectrum. The essential spectrum is always stable, but the discrete spectrum is unstable beyond a critical Marangoni number for a band of wavenumbers in the spanwise direction. Tiwari et al. [11] studied the stability and transient dynamics of thin liquid films flowing over locally heated surfaces. Because the linearized operators governing the evolution of perturbations are nonnormal due to the spatial nonuniformity of the base state, the authors used a nonmodal, transient analysis to determine the maximum amplification of small perturbations to the film.

Frank [12] has performed a numerical simulation of the 3D structure formation by solving the Navier–Stokes equations using the method of particles for incompressible fluid. The formation of periodic rivulet-like structure has been simulated and compared with the experimental results. More recently, Frank and Kabov [13] experimentally and numerically studied the regular structure formation in a film falling down a vertical plate with a built-in rectangular heater. The authors have studied the dependencies of the critical Marangoni number and the wavelength of the most unstable mode on the Reynolds and Weber numbers. Kabova et al. [14] have investigated the influence of the spanwise and streamwise arrangement of the rectangular heaters on 3D structures of the film surface by solving the nonlinear evolution longwave equation.

In the present work, we investigate the influence of mutual location of heaters on the dynamics of a thin film falling down a locally heated plate. The longwave approximation is used to reduce the Navier–Stokes equations with free-surface boundary conditions to a nonlinear partial differential equation for the evolution of the local height of the free surface. We will study two typical cases, i.e. the heaters with rectangular shapes and with infinite spanwise lengths. We solved the 2D and 3D steady states using Newtonian iteration method for these two cases, respectively. Moreover, we studied the linear stability and nonmodal stability of locally heated films for these two typical cases.

The present paper is organized as follows. In Section 2, the mathematic formulation of the physical model is presented. In Section 3, the results and discussions are presented on the problem of steady state. In Section 4, the results and discussions is presented on the linear stability analysis and nonmodal stabilities. In Section 5, we summarize the results and present the conclusions.

2. Mathematical formulation

Consider a thin liquid film falling down an inclined substrate with inclination angle θ with respect to the horizontal direction, as shown in Fig. 1. The coordinate system is constructed with x in the streamwise direction, y the spanwise direction and z normal to the substrate. A heater or a couple of heaters is embedded in the substrate and produces the temperature field $T_0(x,y)$ at the plate surface. Far upstream of the heater, the film has constant thickness, H. The thickness of the film is denoted by $h(t,x,y)$. The velocity field $\mathbf{u}$, temperature T and pressure p of the film are governed by the continuity equation, the Navier–Stokes equation and the energy equation for incompressible Newtonian fluids:

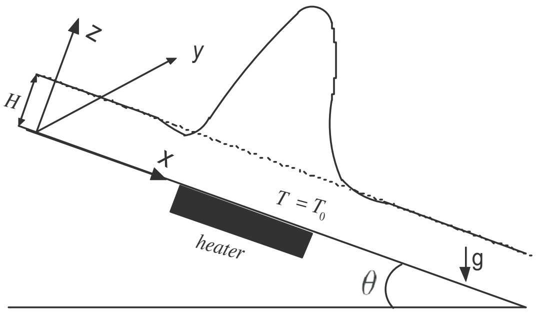

Fig. 1. Sketch of the physical model of a thin film flowing down a locally heated plane.