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Mixed convection stagnation point flow past a vertical flat plate with a second order slip: Heat flux case

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ABSTRACT

Steady mixed convection stagnation point flow past a vertical flat plate with a second order slip when the plate is maintained at a variable heat flux is investigated. By selecting appropriate similarity variables, the partial differential equations are transformed into ordinary (similarity) differential equations, which are then solved numerically using the function bvp4c from Matlab for different values of the governing parameters. It is found that the solutions of the ordinary (similarity) differential equations have two branches, upper and lower branch solutions, in a certain range of the mixed convection and velocity slip parameters. In order to establish which of these solutions are stable and which are not, a stability analysis has been performed. The effects of the mixed convection and velocity slip parameters on the skin friction coefficient, dimensionless wall temperature, and dimensionless velocity and temperature profiles are presented graphically, and discussed in details. It results in that the second order slip affects considerably the flow and heat transfer characteristics.

1. Introduction

The two-dimensional mixed convection stagnation point flow results from a two dimensional flow impinging on a vertical surface at right angles and flowing thereafter symmetrically about the stagnation line. Hiemenz [1] is the first who has studied the steady two-dimensional forced convection stagnation flow using a similarity transformation, to reduce the Navier–Stokes equations to a nonlinear ordinary differential equation. This problem was then extended by Homann [2] to the case of axisymmetric stagnation point flow and by Eckert [3] to the corresponding heat transfer problem. The mixed convection in stagnation flow is important when the buoyancy forces due to the temperature difference between the surface and the free stream become large. Therefore, both the flow and thermal fields are significantly affected by the buoyancy forces. It should be remarked that even though the stagnation point solutions are valid in a small region in the vicinity of the stagnation point of a two or three dimensional body, they represent several physical flows of engineering significance.

In the past several years considerable amount of interest has been given to the free and forced convection stagnation point flows of a viscous fluid. The relevant literature can be found in the books by Gebhart et al. [4], Schlichting and Gersten [5], Pop and Ingham [6], White [7], etc. The usual way in which convective flows are modeled is to assume that the flow is driven either by a prescribed surface temperature (Sparrow and Gregg [8]) or by a prescribed surface heat flux (Wilks [9,10], Carey and Gebhart [11], Merkin and Mahmood [12], Ghosh Moulic and Yao [13,14], Merkin [15]) or by Newtonian heating from the bounding surface (see Merkin [16]) or by convective surface boundary condition (see Aziz [17], Makinde and Olanrewaju [18], and Merkin and Pop [19]).

All these investigations on free and forced convection stagnation point flow and heat transfer are, however, without considering the effect of the velocity slip. Wu [20] proposed a new second order slip velocity model which matches better with the Fukui–Kaneko [21] results based on the direct numerical simulation of the linearized Boltzmann equation [21]. During the last several years, great interest in the flow problems with partial slip has been revealed (Andersson [22], Ariel [23], Wang [24–28], Fang and Lee [29,30]) and Fang et al. [31].

The main purpose of this paper is to extend the work done by Ramachandran et al. [32] for the mixed convection stagnation point flow of a viscous fluid past a vertical flat plate including a second order slip. The case when the imposed heat flux at the surface of the plate varies linearly with the distance along the plate is considered. Using appropriate similarity variables, the partial differential equations are transformed into ordinary (similarity) differential equations, which are then solved numerically. A stability analysis is also performed to show the physically realizable of the dual solutions. It is found that the skin friction or the surface shear stress, the surface temperature, and the velocity and temperature profiles are substantially affected by the second order slip

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Nomenclature

a	first order velocity slip parameter
b	second order velocity slip parameter
A, B	constants
c	constant in the outer flow velocity
C_f	skin friction coefficient
$f(\eta)$	similarity variable
g	acceleration due to gravity
Gr_x	local Grashof number
k	thermal conductivity
K_n	Knudsen number
l	a parameter in the slip velocity model
Nu_x	local Nusselt number
p	pressure
Pr	Prandtl number
q_w	heat flux from the surface of the plate
Re_x	local Reynolds number
t	time
T	temperature of the fluid
T_w	wall temperature
T_∞	ambient temperature
u	velocity component along x-axis
$u_e(x)$	outside boundary layer velocity
$u_{\text{slip}}(x)$	slip velocity
v	velocity component along y-axis
x	coordinate measured along the plate in the vertical direction
y	coordinate measured in the direction normal to the plate

Greek symbols

α	thermal diffusivity
β	coefficient of thermal expansion
δ	molecular mean free path
ε	momentum accommodation coefficient
γ	eigenvalue parameter
η	similarity variable
λ	mixed convection or buoyancy parameter
λ_c	critical mixed convection parameter
μ	dynamic viscosity
ν	kinematic viscosity
$\theta(\eta)$	dimensionless temperature
ρ	density
τ	dimensionless time
τ_w	surface skin friction
ξ	dimensionless distance along the plate
ψ	stream function
ψ	dimensionless stream function

parameters. We mention to this end that Goldstein [33] has called such an opposing flow as a backward boundary layer flow.

2. Basic equations

Fig. 1 shows the geometry of the problem under consideration. It consists of the mixed convection stagnation point flow of a viscous and incompressible fluid over a vertical flat plate coinciding with the plane $y = 0$, the flow being confined to $y > 0$, where y is the coordinate measured in the normal direction to the surface of the plate. It is assumed that the velocity distribution far from the plate (in the inviscid flow) is $u_e(x) = cx$ where x is the coordinate measured along the plate and $c(> 0)$ is a constant. It is also assumed that the heat flux from the plate is $q_w(x)$, while the constant temperature of the ambient fluid is T_∞. Under these conditions along with the Boussinesq approximation, the equations which govern this problem are (see Pop and Ingham [6] or White [7])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + g\beta(T - T_\infty) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

subject to the initial and boundary conditions

$$\begin{aligned} &t < 0: v = 0, \quad u = 0, \quad T = T_\infty \quad \text{for any } x, y \\ &t \geqslant 0: v = 0, \quad u = u_{\text{slip}}(x), \quad -k\tfrac{\partial T}{\partial y} = q_w(x) \quad \text{at } y = 0 \\ &u \to u_e(x), \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{5}$$

where t is the time, u and v are the velocity components along x and y axes, $u_{\text{slip}}(x)$ is the velocity slip, T is the fluid temperature, g is the gravitation acceleration, and the physical meaning of the other quantities is mentioned in the Nomenclature. Further, we assume that $q_w(x)$ and $u_{\text{slip}}(x)$ are given by

$$\begin{aligned} &q_w(x) = q_0 x \\ &u_{\text{slip}}(x) = \frac{2}{3}\left(\frac{3 - \varepsilon l^2}{\varepsilon} - \frac{3}{2}\frac{1 - l^2}{K_n}\right)\delta\frac{\partial u}{\partial y} - \frac{1}{4}\left[l^4 + \frac{2}{K_n^2}(1 - l^2)\right]\delta^2\frac{\partial^2 u}{\partial y^2} = A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2} \end{aligned} \tag{6}$$

where q_0 is the constant heat flux characteristic with $q_0 > 0$ for the assisting flow, and $q_0 < 0$ for the opposing flow, respectively. Further A and B are constants, K_n is Knudsen number, $l = \min(1/K_n, 1)$, ε is the momentum accommodation coefficient with $0 \leqslant \varepsilon \leqslant 1$, and δ is the molecular mean free path. Based on the definition of l, it is seen that for any given value of K_n, we have $0 \leqslant l \leqslant 1$. Since the molecular mean free path δ is always positive it results in that B is a negative number. It should be mentioned that the expression (6) for $u_{\text{slip}}(x)$ has been given by Wu [20] and used also by Fang et al. [31].

3. Steady-state flow case

We look for a similarity solution of Eqs. (1)–(4) with the boundary conditions (5) of the following form

$$\psi = \sqrt{c\nu}\, x f(\eta), \quad \theta(\eta) = (T - T_\infty)/\left[q_w(x)\sqrt{\nu/c}/k\right], \quad \eta = \sqrt{c/\nu}\, y \tag{7}$$

where ψ is the stream function which is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. With these definitions, the velocities u and v are expressed as

$$u = cxf'(\eta), \quad v = -\sqrt{c\nu}\, f(\eta), \tag{8}$$

where prime denotes differentiation with respect to η. The pressure p can be derived from Eq. (3) and is given by