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Second law analysis for Poiseuille flow of immiscible micropolar fluids in a channel

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ABSTRACT

In this paper, the problem of steady Poiseuille flow of two immiscible incompressible micropolar fluids between two horizontal parallel plates of a channel with constant wall temperatures is studied in terms of entropy generation. The flow is assumed to be governed by Eringen's micropolar fluid flow equations. The flow region is divided into two zones, the flow of the heavier fluid taking place in the lower zone-I. No slip condition is taken on the plates and at the interface continuity of velocity, micro-rotation, temperature, heat flux and shear stresses is imposed. The velocity, micro-rotation and temperature fields are derived analytically. The dimensionless quantities-entropy generation number (*Ns*), Bejan number (*Be*) and irreversibility ratio (ϕ) are analytically derived. The effects of material parameters like micropolarity (c_i), couplestress (s_i) on the velocity, micro-rotation and temperature are investigated. The derived equation for the dimensionless entropy generation number is used to interpret the relative importance of frictions to conduction by varying viscous dissipation parameter. The entropy generation near the plates increases more rapidly in fluid I than in fluid II as viscous dissipation effects become more important in zone I. The velocity and temperature profiles are found to be in good agreement with the distributions of the dimensionless entropy generation number (*Ns*).

1. Introduction

There is a great demand in many industries and projects to thoroughly analyze, improve and design the power systems. In classical methods, the efficiency of the power systems is studied based on first law of thermodynamics. The recent methodologies study the systems based on second law of thermodynamics. The new methodology is called exergy analysis (analysis of available work). In heat transfer process in any system involves exergy losses i.e., destroy the available work due to temperature gradients and fluid frictions. This is due to irreversible work involved in the process. This can be accounted by second law of thermodynamics. Exergy loss is proportional to entropy generation rate. Hence minimization of entropy generation rate indicates optimum exergy or amount of available work. These methods are popularly known as Entropy Generation Minimization (EGM) methods. This was first introduced by Bejan [1,2] and he gave good engineering sense for the study by focusing on irreversibility. This new methodology is based on simultaneous application of first and second law of thermodynamics in analysis and design of the systems. Bejan [1] studied the heat transfer problems in the pipe flow, boundary layer flow past a plate, flow in the entrance region of a rectangular duct using EGM. Bejan [3] demonstrated how the difference between reversible work and work is proportional to entropy generation rate. In the paper he explained how EGM is useful in obtaining optimal allocation of heat transfer area, optimal latent heat storage temperature and optimal sensible heat storage time interval. These methods can be found in detail in the treatises by Bejan [4–7] and Bejan et al. [8].

The flow and heat transfer of immiscible fluids are of special importance in the petroleum extraction and transport problem. The reservoir rock of oil field contains many immiscible fluids in its pores. A portion of the pores contains water and the rest contains oil or gases or both. The immiscible flows in crude oil transport was studied experimentally by Bakhtiyarov et al. [9]. Oscillatory flow and heat transfer in two immiscible viscous fluids was examined analytically by Chamkha [10]. Kamisli et al. [11] explained very nicely the thermodynamic interface conditions involved in a flow of immiscible fluids. They observed that minimum temperature gradient in transverse direction of the flow offers minimum entropy generation near the plates. variation of irreversibility in terms of Bejan number (*Be*) and energy stream line tracking inside a porous channel are explained in detail by

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Nomenclature

Be	Bejan number $\left(=\frac{1}{1+\phi}\right)$
Br	Brinkman number $(=Ek\,Pr)$
$\frac{Br}{\Omega}$	viscous dissipation parameter
$c_i=\frac{\kappa_i}{\mu_i}$	material parameter or micropolarity parameter
C	non-dimensional micro-rotation component in z-direction
d_{ij}	components of the strain
D	deformation tensor
E	specific internal energy
Ek	Eckert number
$\bar{f}$	body forces per unit mass
2h	height of the free channel
$\bar{\mathbf{h}}$	heat flux
j	gyration coefficient
k_1, k_2	thermal conductivity of the fluid in zones-I, II
$\bar{l}$	body couple per unit mass
m_{ij}	couple stress tensor
n_β	ratio of couple stress viscosity coefficients $\left(=\frac{\beta_2}{\beta_1}\right)$
n_k	ratio of thermal conductivities $\left(=\frac{k_2}{k_1}\right)$
n_μ	ratio of viscosities $\left(=\frac{\mu_2}{\mu_1}\right)$
n_ρ	ratio of densities $\left(=\frac{\rho_2}{\rho_1}\right)$
Nf_i	entropy generation due to viscous dissipation
Ns_i	dimensionless total entropy generation number
Ny_i	entropy generation due to transverse conduction
Nu	Nusselt number
P	pressure
Pr	Prandtl number
$\bar{q}$	velocity vector
Re	Reynolds number
s_1,s_2	couple stress parameters
$(S_i)_G$	entropy generation rate
$(S_i)_{G,C}$	characteristic entropy transfer rate
t_{ij}	stress tensor
T_1, T_2	non-dimensional temperatures of the plates
u	non-dimensional velocity in X-direction
x, y	non-dimensional space coordinates
X, Y	space co-ordinates

Greek symbols

α, β, γ	gyration viscosity coefficients
δ_{ij}	kronecker delta
δ_1	couple stress parameter $\left(=\frac{\beta_1}{\mu_1 h^2}\right)$
ϵ_{ijk}	Levi–Civita symbol or permutation symbol
κ_1, κ_2	micro-rotation viscosity coefficients
μ_1, μ_2	viscosity coefficients
$\bar{v}$	micro-rotation vector
Ω	dimensionless temperature difference $\left(=\frac{\Delta T}{T_o}\right)$
Φ	dissipation function
ϕ	irreversibility distribution ratio $\left(=\frac{Nf}{Ny}\right)$
ρ	density
θ	non-dimensional temperature

Subscripts

1	fluid in zone I
2	fluid in zone II

Shohel Mahmud et al. [12]. The effect of geometric parameters to find optimum shape of the ducts by using second law analysis is studied by Sahin [13–15] and Hakan [16].

This paper aims at second law analysis for the flows of two immiscible micropolar fluids in a parallel plate channel. Micropolar fluids exhibit couple stresses and the particles of the fluid have independent rotation vector in addition to velocity vector. This theory of micropolar fluids was proposed by Eringen [17,18]. For experimental determination of parameters of micropolar fluids one can refer Migun et al. [19] and Kolpashchikov et al. [20]. An account of the earlier developments in polar fluid theory can be found in the book by V.K.Stokes [21]. Some basic viscous flows in micropolar fluids was discussed by Ariman et al. [22] and the existing state of art can be seen in the excellent treatise of Lukaszewicz [23]. Jerome et al. [24] gave molecular interpretation for the Poiseuille flow of a micropolar fluid.

The problem of simultaneous flow of immiscible fluids in channels is of importance in industrial processes such as transportation of two or more fluids in the same pipe or channel. So there has been widespread interest in the study of flow through channel and tubes in the recent years. In many of the areas fluid flow, flow of immiscible liquids or multi-phase fluids occur. For example blood flow in arteries has been studied by many researchers considering blood as two phase flow [25]. In view of these, several investigations on multiphase flows are reported by various researchers. Bird et al. [26] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Kapur et al. [27] have studied the flow of two immiscible incompressible viscous fluids between two parallel plates. Bhattacharya [28] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Mass transfer into a laminar fluid stream from the moving interface of two immiscible fluids between parallel plates was discussed by Hikita et al.[29]. Jie Li et al. [30] have discussed numerical study of flows of two immiscible liquids at low Reynolds number. Chamka et al. [31] discussed flow of two immiscible fluids in a porous and non-porous channel. Malashetty et al. [32] have discussed the convective magnetohydrodynamic two fluid flow and heat transfer in an inclined channel. Umavathi et al. [33] studied unsteady two-fluid flow and heat transfer in a horizontal channel. Prathap Kumar et al. [34] analytically examined fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel. Dragis Nikodijevic et al. [35] have studied MHD Couette two-fluid flow and heat transfer in presence of uniform inclined magnetic field. The heat transfer of two immiscible fluids in the presence of uniform inclined magnetic field was discussed by Nikodijevic et al. [36]. Szeri et al. [37] discussed the flow of a non-Newtonian fluid between heated parallel plates. Nield et al. [38] discussed thermally developing forced convection in a porous medium between two parallel plates with walls maintained at uniform temperature.

The present study is taken up in view of realistic situations cited in [9,25] and growing importance of study of entropy generation methods (EGM). Many researchers considered the immiscible flow of viscous fluids. But very few [33] have taken up the study of micropolar fluids. Since micropolar fluids represent the more general and realistic study of properties of crude oils, blood, etc. Here we are considering the flow of immiscible micropolar fluids between the parallel plates.

2. Fomulation of the problem

The physical model of the flow shown in Fig. 1, consists of two parallel plates extending in the X-direction. The height between the plates is 2h. The plates are maintained at constant temperatures. The width of the plates is much greater than the distance be-