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A hybrid transform technique for the hyperbolic heat conduction problems

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ABSTRACT

In this present study, we propose a hybrid transform technique to investigate the hyperbolic heat conduction problem. The Laplace transform is used to remove the time-dependent terms from the governing equation and the s-domain dimensionless temperature function is obtained by the integral transform scheme. Finally, the analytical solution of this problem can be determined by taking the inverse integral transform and inverse the Laplace transform. In this paper, six different examples have been solved by the present technique. It is shown that the analytical solutions of six hyperbolic heat conduction problems can be obtained by this technique.

1. Introduction

In the past, the Fourier heat conduction equation gives quite excellent approximations for most engineering applications. But the theory becomes unsuitable when we are interested in transient heat flow in an extremely short period of time or at very low temperatures, such as in laser-aided material processing, cryogenic engineering, and the high-intensity electromagnetic irradiation at a solid. Thus, it is necessary to modify Fourier heat flux model for these problems that account for phenomena involving finite propagation speed of thermal wave. The description of hyperbolic heat conduction process can be found Refs. [1–3]. The one-dimensional analytical solutions of hyperbolic heat conduction can be found in a number of publications such as Tsai-tse Kaov [4] studied the problem in thin surface layers. Baumeiser et al. [5] studied for a semi-infinite medium problem. Taitel [6] investigated the problem in a finite medium with convection. Ozisik et al. [7] and Wu [8] studied for the radiation at the wall surface.

Carey et al. [9] applies the central and backward difference schemes to examine the oscillation of numerical solution at the reflected boundary. And many numerical schemes have been proposed to remedy the numerical oscillation at the wave front such as the predictor–corrector scheme [10], the transfinite element formulation [11], a technique based on the Galerkin finite element and mixed implicit–explicit scheme [12], the characteristic method [13], and the hybrid scheme [14].

The effect of the surface radiation on thermal wave propagation in a one-dimensional slab has been studied by Glass et al. [15] and Yeung et al. [16]. And two-dimensional problem is studied by Yang [17], Chen and Lin [18] and Shen [19]. The problem in thin surface layers has been investigated by Chen [20]. Loh et al. [21] study the comparison of the solution using the Fourier and Non-Fourier law in the heat conduction problem. Chen [22] proposes the hybrid Green's function method to study the three dimensional problem.

The purpose of the present study is to propose a hybrid transform technique investigating the hyperbolic heat conduction problem. The Laplace transform method is used to remove the time-dependent terms from the governing equation, and then the s-domain dimensionless temperature function is obtained by using an integral transform scheme. Finally, the time-domain dimensionless temperature can be obtained by taking the inverse integral transform and inverse Laplace transform. From these examples in the paper, it is shown that the technique can be obtained the analytical solution of hyperbolic heat conduction problem.

2. Analysis

Consider the hyperbolic heat conduction problem. The hyperbolic heat conduction equation is given as

$$\frac{1}{C^2}\frac{\partial^2 T}{\partial t^2}+\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(\mathbf{r},t)}{k}+\frac{\tau}{k}\frac{\partial g(\mathbf{r},t)}{\partial t} \quad (1)$$

with initial condition being

$$T(\mathbf{r},t)=F(\mathbf{r}),\quad \frac{\partial T}{\partial t}(\mathbf{r},t)=K(\mathbf{r})\quad \text{for } t=0 \quad (2)$$

For convenience of analysis, let us define by the following dimensionless variables

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Nomenclature

A	s-domain function
B	real number
C	propagation velocity of thermal wave (m/s)
c_p	specific heat (KJ/kg K)
F	initial conduction function (K)
$\widehat{F}$	dimensionless initial conduction function
f_r	reference heat flux (KJ)
g	heat source (KJ/m^3)
G	dimensionless heat source, $\frac{4\alpha g}{Cf_r k}$
H	Heaviside function
K	initial conduction function (K/s)
$\widehat{K}$	dimensionless initial conduction function
k	thermal conductivity (KJ/m^2 K)
l	number continuous boundary surfaces
L	length (m)
m	integer number
N_η	η-directional normalization integral, $\int_{R_\eta}[\phi_\eta(\lambda_m,\eta')]^2 d\eta'$
N_ζ	ζ-directional normalization integral, $\int_{R_\zeta}[\phi_\zeta(\alpha_p,\zeta\prime)]^2 d\zeta'$
N_ς	ς-directional normalization integral, $\int_{R_\varsigma}[\phi_\varsigma(\beta_n,\varsigma')]^2 d\varsigma'$
n	integer number
$\vec{n}$	outward-drawn normal vector to the boundary surface
p	integer number
q	heat flux (KJ)
Q	dimensionless heat flux, $\frac{q}{f_r}$
R	region of system
r	space variable (m)
S_i	boundary surface of system
s	Laplace transform parameter
T	temperature (K)
T_0	surrounding temperature (K)
x, y, z	coordinate (m)
Greek letters	
α	thermal diffusivity, $\frac{k}{\rho c_p}$
α_p	ζ-directional eigenvalue
β_n	ς-directional eigenvalue
η	dimensionless coordinate, $\frac{Cx}{2\alpha}$
θ	dimensionless temperature, $\frac{(T-T_0)kC}{\alpha f_r}$
λ_m	η-directional eigenvalue
$\varphi_\eta(\lambda_m,\eta)$	η-directional eigenfunction
$\varphi_\zeta(\alpha_p,\zeta)$	ζ-directional eigenfunction
$\varphi_\eta(\beta_n,\varsigma)$	ς-directional eigenfunction
ρ	density (Kg/m^3)
ς	dimensionless coordinate, $\frac{Cz}{2\alpha}$
ξ	dimensionless time, $\frac{C^2 t}{2\alpha}$
ζ	dimensionless coordinate, $\frac{Cy}{2\alpha}$
Superscript	
-	the Laplace transform
$\cap$	dimensionless form
$\wedge$	integral transform

$$\xi = \frac{C^2 t}{2\alpha} \tag{3}$$

$$\eta = \frac{Cx}{2\alpha} \quad \zeta = \frac{Cy}{2\alpha} \quad \varsigma = \frac{Cz}{2\alpha} \tag{4}$$

$$\theta(\eta,\xi) = \frac{kC(T-T_0)}{\alpha f_r} \tag{5}$$

$$G = \frac{4\alpha g}{Cf_r k} \tag{6}$$

The resulting equation becomes

$$\frac{\partial^2\theta}{\partial\xi^2} + 2\frac{\partial\theta}{\partial\xi} = \frac{\partial^2\theta}{\partial\eta^2} + \frac{\partial^2\theta}{\partial\zeta^2} + \frac{\partial^2\theta}{\partial\varsigma^2} + \frac{1}{2}\frac{\partial G}{\partial\xi} + G \tag{7}$$

with initial condition

$$\theta(\mathbf{r},\xi) = \widehat{F}\,\frac{\partial\theta}{\partial\xi}(\mathbf{r},\xi) = \widehat{K} \quad \text{for } \xi = 0 \tag{8}$$

2.1. The hybrid transform technique

To remove the ξ-dependent terms, take the Laplace transform of Eq. (7) with respect to ξ, giving

$$\frac{\partial^2\overline{\theta}}{\partial\eta^2} + \frac{\partial^2\overline{\theta}}{\partial\zeta^2} + \frac{\partial^2\overline{\theta}}{\partial\varsigma^2} + \left(\frac{s}{2}+1\right)\overline{G} - (s^2+2s)\overline{\theta} + (s+2)\,\widehat{F} + \widehat{K} \tag{9}$$

To solve the above s-domain heat conduction problem with boundary condition

$$\frac{\partial\overline{\theta}}{\partial\vec{n}} + B\overline{\theta} = \bar{f}_i \quad \text{on } S_i \tag{10}$$

We consider the following integral transform pair [23]Inversion formula:

$$\overline{\theta}(\eta,\zeta,\varsigma,s) = \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{n=1}^{\infty}\frac{\varphi_\eta(\lambda_m,\eta)}{N_\eta(\lambda_m)}\,\frac{\varphi_\zeta(\alpha_p,\zeta)}{N_\zeta(\alpha_p)} \times \frac{\varphi_\varsigma(\beta_n,\varsigma)}{N_\varsigma(\beta_n)}\hat{\overline{\theta}}(\lambda_m,\alpha_p,\beta_n,\eta,\zeta,\varsigma,s) \tag{11}$$

Integral transform:

$$\hat{\overline{\theta}}(\lambda_m,\alpha_p,\beta_n,s) = \int_{R_\eta}\int_{R_\zeta}\int_{R_\varsigma}\varphi_\eta(\lambda_m,\eta')\varphi_\zeta(\alpha_p,\zeta')\varphi_\varsigma(\beta_n,\varsigma') \times \overline{\theta}(\eta',\zeta',\varsigma',s)\,d\eta' d\zeta' d\varsigma' \tag{12}$$

where $\varphi_\eta(\lambda_m,\eta)$, $\varphi_\zeta(\alpha_p,\zeta)$, and $\phi_\varsigma(\beta_n,\varsigma)$ are the eigenfunction and $N_\eta(\lambda_m)$, $N_\zeta(\alpha_p)$, and $N_\varsigma(\beta_n)$ are the normalization integrals.

$$N_\eta(\lambda_m) = \int_{R_\eta}[\varphi_\eta(\lambda_m,\eta')]^2 d\eta', \quad N_\zeta(\alpha_p) = \int_{R_\zeta}[\varphi_\zeta(\alpha_p,\zeta')]^2 d\zeta' \quad \text{and } N_\varsigma(\beta_n) = \int_{R_\varsigma}[\varphi_\varsigma(\beta_n,\varsigma')]^2 d\varsigma' \tag{13}$$

The solution of Eq. (9) subject to the boundary conditions (10) is given as

$$\overline{\theta}(\eta,\zeta,\varsigma,s) = \sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{[N_\eta(\lambda_m)+N_\zeta(\alpha_p)+N_\varsigma(\beta_n)+s^2+2s]} \cdot \varphi_\eta(\lambda_m,\eta)\varphi_\zeta(\alpha_p,\zeta)\varphi_\varsigma(\beta_n,\varsigma) \left[(s+2)\hat{\widehat{F}}(\lambda_m,\alpha_p,\beta_n) + \hat{\widehat{F}}(\lambda_m,\alpha_p,\beta_n) + A(\lambda_m,\alpha_p,\beta_n,s)\right] \tag{14}$$

where

$$\hat{\widehat{F}}(\lambda_m,\alpha_p,\beta_n) = \int_{R_\eta}\int_{R_\zeta} \times \int_{R_\varsigma}\varphi_\eta(\lambda_m,\eta')\varphi_\zeta(\alpha_p,\zeta')\varphi_\varsigma(\beta_n,\varsigma')\hat{\widehat{F}}(\eta',\zeta',\varsigma')\,d\eta' d\zeta' d\varsigma' \tag{15}$$