Electric Power Systems Research 78 (2008) 1672–1678

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Some analytical results pertaining to Cournot models for short-term electricity markets

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ARTICLE INFO

Article history:
Received 1 June 2007
Received in revised form 29 January 2008
Accepted 24 February 2008
Available online 8 April 2008

Keywords:
Cournot model
Market equilibrium
Analytical results
Sensitivities

ABSTRACT

This paper provides some theoretical results pertaining to the Cournot model applied to short-term electricity markets. Price, quantities and profits are first obtained, and then results related to sensitivities and limit values are derived and discussed. The cases of both several identical Cournot producers and one dominant Cournot producer are analyzed. A case example illustrates the results obtained.

1. Introduction

1.1. Background and aim

The well-known Cournot model is often used to describe the behavior of generating companies (gencos) in electricity markets. However, no comprehensive analytical description of such model is available in the context of an electricity market environment. This paper gives a tutorial review of this model and provides some contributions pertaining to the computation and interpretation of sensitivities and limit values.

We consider n gencos, assume that each genco uses a Cournot model to derive its involvement in the market, and obtain some relevant analytical results characterizing the market equilibrium and related to sensitivities and limit values. It is considered that the transmission network does not influence the equilibrium, i.e., that no network constraint is binding.

1.2. Literature review

The Cournot model is well established in the microeconomics literature; see, for instance, the classical manuals by Varian [1], Mas-Colell et al. [2] or Nicholson and Snyder [3]. To analyze electricity markets, Cournot models are often encountered in the technical literature as they adequately represent producer behavior in real-world markets. Relevant references using Cournot models within an electricity market framework include [4–18].

In Ref. [4], the possible strategic interactions within an electricity market between Cournot and competitive producers are discussed and illustrated. Market power issues are addressed. Ref. [5] describes a network-constrained Nash equilibrium assuming that producers behave in a Cournot manner and are subject to regulated transmission prices. Ref. [6] uses a linear complementarity program (LCP) to computationally analyze and characterize Cournot-based equilibria in both a pool and a bilateral contracting market. Ref. [7] considers the generation expansion planning problem and uses Cournot models to characterize the behavior of the producers. A comprehensive numerical study is carried out. Ref. [8] analyzes, explores, discusses and illustrates network-constrained Cournot equilibria using small networks with few producers/consumers. In Ref. [9], a conjectured supply function approach is numerically compared with the classical Cournot approach. Appropriate conclusions are duly drawn. In Ref. [10], Cournot and supply function equilibria are numerically compared under transmission constraints. Different cases are presented and discussed. Ref. [11] proposes a model of Nash–Cournot competition including a linearized dc network with arbitrage. A rich discussion is embedded in the cases studied. Ref. [12] considers simultaneously diverse electricity markets (pool, bilateral contracting, etc.) that are jointly analyzed and characterized under a Cournot framework. Ref. [13] analyzes the electricity market equilibrium for Cournot agents and proposes an algorithmic approach to characterize such equilibrium. It uses analytical equations similar to some of the formulae derived in this paper. Ref. [14] proposes and characterizes a model of Nash–Cournot competition including piecewise

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doi:10.1016/j.epsr.2008.02.008

linear demand functions. In Ref. [15], the Cournot equilibrium is computationally analyzed under uncertainty pertaining to both generator availability and demand. Ref. [16] provides an experimental economics approach to analyze an electricity market with Cournot agents. A diversity of realistic experiments are carried out and conclusions duly drawn. Recent Refs. [17,18] provide and illustrate an algorithm to identify all Nash–Cournot equilibria under some simplifying assumptions.

Complementing the above approaches, which are computationally oriented, this paper provides close formulae characterizing analytically the Cournot equilibrium in a short-term electricity market. It expands some results provided in Ref. [13] and complements the fully experimental approach provided in Ref. [16].

1.3. Contributions

The contributions of this paper are threefold:

1. Providing general formulae for the Cournot equilibrium in electricity markets.
2. Providing sensitivity results of a Cournot equilibrium within an electricity market.
3. Providing limit values characterizing the Cournot equilibrium if the slope of the price/quantity curve approaches 0.
4. Providing a comparison between the case of several identical Cournot gencos and the case of one dominant Cournot genco.

1.4. Paper organization

This paper is organized as follows. Section 2 derives general results including price, productions and profits, as well as sensitivity and limit values. Section 3 considers the case of several identical Cournot gencos, and Section 4 the case of one dominant Cournot genco. Section 5 compares the cases of (i) identical gencos and (ii) one dominant genco. Section 6 provides and discusses results from a case example. Section 7 provides some conclusions.

2. Cournot model

Seeking maximum profit, a Cournot genco assumes that its production affects the market-clearing price, but not the productions of its competing gencos.

Each genco is characterized by a power portfolio with associated production cost given by the quadratic function:

$$C_i(q_i) = a_i + b_i \cdot q_i + \frac{1}{2} c_i \cdot q_i^2, \tag{1}$$

where $a_i > 0$, $b_i > 0$ and $c_i > 0$ for this cost function to be well behaved. The number of gencos considered is n.

A linear price/demand curve is considered, i.e.

$$p(q_1 + \cdots + q_n) = \gamma + \beta \cdot (q_1 + \cdots + q_n) \quad \text{with } \gamma > 0 \text{ and } \beta < 0. \tag{2}$$

Conditions $\gamma > 0$ and $\beta < 0$ are required for the price/demand curve to be well defined. For a limit-value analysis, additionally, we explicitly recognize that $|\beta|$ is small and tends to 0.

A linear price/demand curve embodies an appropriate tradeoff between modeling accuracy and analytical complexity.

To be able to derive analytical expressions, no network constraint is considered to be binding, i.e., the network is not explicitly modeled.

2.1. Formulation

The profit maximization problem for genco i is

$$\text{Maximize}_{p,q_i} \quad \pi_i = p(q_1 + \cdots + q_n) \cdot q_i - C_i(q_i). \tag{3}$$

Considering the price/demand function (2) and solving simultaneously the profit maximization problem (3) of each genco (market equilibrium), the results in the section below are obtained.

2.2. Solution

Imposing simultaneously $\partial \pi_i / \partial q_i = 0$ for all gencos, and taking into account (1) and (2), we obtain the results below for the market-clearing price, the quantities produced and the profits obtained.

The resulting market-clearing price is

$$p = \frac{(\gamma/\beta) - \sum_{i=1}^{n} (b_i/(c_i - \beta))}{(1/\beta) - \sum_{i=1}^{n} (1/(c_i - \beta))}. \tag{4}$$

Since $\beta < 0$, $\gamma > 0$, $b_i > 0$ and $c_i > 0$, the equilibrium price provided by (4) is always positive.

The quantity produced by each genco is

$$q_i = \frac{((\gamma - b_i)/\beta) + \sum_{j=1}^{n} ((b_i - b_j)/(c_j - \beta))}{\left((1/\beta) - \sum_{j=1}^{n} (1/(c_j - \beta))\right) \cdot (c_i - \beta)}, \quad i = 1, \ldots, n, \tag{5}$$

which is positive if the numerator of (5) is negative (the denominator is always negative), i.e., if $\gamma - b_i + \beta \sum_{j=1}^{n} ((b_i - b_j)/(c_j - \beta)) > 0$.

It should be noted that both the price and the produced quantities do not depend on the no-load cost parameters a_i.

Finally, the profit obtained by each genco is

$$\pi_i = \left(\frac{((\gamma - b_i)/\beta) + \sum_{j=1}^{n} ((b_i - b_j)/(c_j - \beta))}{\left((1/\beta) - \sum_{j=1}^{n} (1/(c_j - \beta))\right) \cdot (c_i - \beta)} \right)^2 \cdot \left(\frac{c_i}{2} - \beta \right) - a_i, \quad i = 1, \ldots, n. \tag{6}$$

2.3. Solution in compact form

The classical results presented in Section 2.2 above can be expressed in a compact manner. Defining the three $(n+1) \times 1$ vectors below

$$\boldsymbol{\sigma} = \begin{pmatrix} \gamma \\ b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, \quad \boldsymbol{\tau} = \begin{pmatrix} -\dfrac{1}{\beta} \\ \dfrac{1}{c_1 - \beta} \\ \dfrac{1}{c_2 - \beta} \\ \vdots \\ \dfrac{1}{c_n - \beta} \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \tag{7}$$

we can write (4)–(6) in a compact manner. Expressions for price, quantities and profits become

$$p = \frac{\boldsymbol{\sigma}^{\mathrm{T}} \bullet \boldsymbol{\tau}}{\mathbf{e}^{\mathrm{T}} \bullet \boldsymbol{\tau}}, \tag{8}$$

$$q_i = -\frac{(b_i \cdot \mathbf{e}^{\mathrm{T}} - \boldsymbol{\sigma}^{\mathrm{T}}) \bullet \boldsymbol{\tau}}{(c_i - \beta) \cdot \mathbf{e}^{\mathrm{T}} \bullet \boldsymbol{\tau}}, \quad i = 1, \ldots, n, \tag{9}$$

$$\pi_i = \left(-\frac{(b_i \cdot \mathbf{e}^{\mathrm{T}} - \boldsymbol{\sigma}^{\mathrm{T}}) \bullet \boldsymbol{\tau}}{(c_i - \beta) \cdot \mathbf{e}^{\mathrm{T}} \bullet \boldsymbol{\tau}} \right)^2 \cdot \left(\frac{c_i}{2} - \beta \right) - a_i, \quad i = 1, \ldots, n, \tag{10}$$

where the symbol "•" indicates inner product.

2.4. Limit values

We consider in this section that $\beta \to 0$, i.e., that the demand becomes more and more elastic. Although electricity demand is clearly inelastic, this section provides insights for the case of