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A new formulation for thermal analysis of composites by hybrid boundary node method

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ABSTRACT

In this paper, a new formulation based on the hybrid boundary node method (Hybrid BNM) is proposed for thermal analysis of composites. The Hybrid BNM is a boundary type meshless method which based on the modified variational principle and the Moving Least Squares (MLS) approximation. In the new formulation, continuity conditions are used as the conventional multi-domain solver and the unknowns of the interfaces are assembled only once in the final system equation, which can reduce both the computational time and memory required. The new formulation is quiet suitable for the inclusion-based composites, especially for the case when the inclusions are solid and totally embedded in the matrix domain. The carbon nanotubes (CNTs) based composites are also discussed and studied by the new formulation. It shows that the thickness of the CNT has little influence on the thermal properties of the composites. Numerical examples are presented to verify the new formulation and the results have shown the accuracy and efficiency of the new formulation.

1. Introduction

The heat flow in a composite material is a complex problem because of the anisotropy of material properties. Much effort, time and expense would be saved if the properties of the reinforced composites could be predicted accurately. Numerical methods play an important role in the thermal analysis of composites. Finite element method (FEM) is one of the numerical methods and some models based on it have been proposed so far. Alexander and Tzeng [1] developed a FEM model, which can provide accurate 3D effective properties for arbitrarily shaped 3D continuum elements containing multiple layers of arbitrary orientation and shape. Application of the FEM in predicting the effective transverse thermal conductivity of fiber reinforced composites was systematically studied by Islam and Pramila [2].

Boundary element method (BEM) is another powerful method for the analysis of thermal problem of composites and has been investigated by many researchers in recent years. The continuum models of carbon nanotubes (CNTs) based composites by using the BEM were discussed by Liu and Chen [3]. They proposed 3D models for modeling the CNTs embedded in a matrix and three kinds of representative volume elements (RVEs) namely cylindrical RVE, square RVE and hexagonal RVE for the modeling of CNT based composites were proposed. Nishimura and Liu [4] applied the boundary integral equation (BIE) method for the thermal analysis of fiber-reinforced composite based on a rigid-line inclusion model. The rigid line approximation is valid when the fibers have much higher values of thermal conductivity compared with the matrix. An efficient and simplified BEM formulation for steady-state heat conduction analysis of 3D solids with fiber inclusions was developed by Chatterjee et al. [5]. In their work, for efficient analysis and modeling, the cylindrical shaped fibers were idealized by a system of curvilinear line elements with a prescribed diameter.

Although FEM and BEM have been employed to many areas, it may not be convenience and efficient for them to analyze composites because of the cells caused by the mesh procedure and the complex characteristic of the composites. Meshless methods are widely investigated to avoid the task of mesh generation cause by FEM or BEM. In addition, they are very suitable for solving problems involving changing domains such as large deformation or crack propagation. Many kinds of meshless methods have been proposed so far, including the element free Galerkin method (EFG) [6], the meshless local Petrov–Galerkin (MLPG) approach [7], the boundary node method (BNM) [8], the Galerkin boundary node method (GBNM) [9], the boundary face method (BFM) [10] and the hybrid boundary node method (Hybrid BNM), etc.

The Hybrid BNM was proposed by Zhang et al. [11,12], which combines the MLS approximation scheme with the hybrid displacement variational formula. This method has been developed by Miao et al. [13,14] and applied to elastodynamics problems [15], Helmholtz problems [16] and multi-domain problems [17]. The Hybrid BNM not only reduce the spatial dimensions by one like BEM or BNM, but also does not require boundary element meshes,

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neither for the purpose of interpolation of the solution variables nor for the integration of energy. In fact, the Hybrid BNM requires only discrete node located on the surface of the domain and its parametric representation. As the parametric representation of created geometry is used in most of CAD packages, it should be possible to exploit their Open Architecture feature, and the required coefficients (representation) can be obtained automatically. Thus, it can reduce the human-labor cost for meshing the domain and can be applied to moving boundaries problems and crack propagation problems efficiently.

Few meshless methods have been applied to the analysis of composites so far. Zhang et al. [18] used a simplified model based on the Hybrid BNM for the heat conduction analysis of CNT based nano-composites. In the simplified model, the host polymer is the only domain which is modeled, while the CNTs are treated as heat superconductors with constant and unknown temperatures constrained at their surfaces. They also coupled the method with fast multipole method [19] to large scale analysis of thermal properties of CNT composites [20,21]. Singh et al. [22] applied the EFG to the thermal analysis of CNT based composites by using the cylindrical RVE. In their work, both the mulit-domain and simplified approach were employed and the CNT was also assumed at a constant temperature in the simplified approach. A nonlinear heat conduction analysis of nano-composites by EFG has also been carried out by Singh et al. [23]. In the work of Zhang and Singh, the simplified approach treats the CNTs as heat superconductors and it is especially suitable for the case when the heat conductivities of the inclusions are much higher than the host polymer. However, this approach can not be applied to general composites, such as for the case when the difference of the heat conductivities between the inclusion and matrix are not too much, even though it can reduce the total degrees of freedom (DOFs) substantially. Multi-domain approach can be used to overcome this problem as the matrix and inclusions are modeled as separate regions. However, it leads much more computational time than the simplified model, since the total DOFs containing both the unknowns in matrix and inclusions.

In this paper, a new formulation using the Hybrid BNM is proposed for the thermal analysis of 3D solids with inclusions. The new formulation is based on a multi-domain approach [17,24] and can be applied to general composites like the conventional multi-domain model. It can also reduce the total DOFs as the simplified approach since the unknowns on the interfaces are computed only once while solving the final system equation.

This paper is organized as follows. The multi-domain formulation for Hybrid BNM is reviewed in the second section. In the third section, the new formulation for the thermal analysis of matrix with inclusions is proposed. Finally, in the fourth section, numerical examples are given. The results show that the new formulation is valid and efficient.

2. The multi-domain solver of Hybrid BNM

In this section, the multi-domain solver for composite materials by Hybrid BNM is introduced, which is obtained by assembling the equations for each single domain into an overall system equation by using the continuous and equilibrium relations on the interfaces between the sub-domains.

A steady state heat conduction problem is governed by Laplace's equation with proper boundary conditions. The Hybrid BNM is based on a modified variational principle. In 3D heat conduction problems, the functions in the modified variational principle assumed to be independed are: temperature ϕ inside the domain, boundary temperature $\tilde{\phi}$ and boundary normal heat flux $\tilde{q}$.

Suppose that the problem has n sub-domains and N_i nodes are distributed on the bounding surfaces of subdomain-i. The temperature $\tilde{\phi}$ and normal heat flux $\tilde{q}$ at the boundary are approximated by the MLS approximation as follows:

$$\tilde{\phi}(\mathbf{s}) = \sum_{J=1}^{N_i} \Phi_J(\mathbf{s})\hat{\phi}_J \tag{1}$$

$$\tilde{q}(\mathbf{s}) = \sum_{J=1}^{N_i} \Phi_J(\mathbf{s})\hat{q}_J \tag{2}$$

where $\hat{\phi}_J$ and $\hat{q}_J$ are nodal values, and $\Phi_J(\mathbf{s})$ is the shape function of the MLS approximation, corresponding to node $\mathbf{s}_J$.

The temperature inside the domain can be approximated by fundamental solutions as

$$\phi = \sum_{J=1}^{N_i} \phi_J^s x_J \tag{3}$$

and hence at a boundary point, the normal heat flux is given by

$$q = \sum_{J=1}^{N_i} q_J^s x_J \tag{4}$$

where ϕ_J^s and q_J^s are the fundamental solutions; and x_J are unknown parameters. For 3D steady state heat conduction problems, the fundamental solutions are written as

$$\phi_J^s = \frac{1}{\kappa}\frac{1}{4\pi r(Q, \mathbf{s}_J)} \tag{5}$$

$$q_J^s = -\kappa \frac{\partial \phi_J^s}{\partial n} \tag{6}$$

where κ is the thermal conductivity, Q is a field point and $r(Q, \mathbf{s}_J)$ is the distance between the point Q and the node $\mathbf{s}_J$.

Using the modified functional variational principle in all local-regions around the boundary nodes, the following set of Hybrid BNM for subdomain-i can be written as

$$\mathbf{U}^i\mathbf{x}^i = \mathbf{H}^i\hat{\boldsymbol{\phi}}^i \tag{7}$$

$$\mathbf{V}^i\mathbf{x}^i = \mathbf{H}^i\hat{\mathbf{q}}^i \tag{8}$$

In the above equations, the elements of $\mathbf{U}$, $\mathbf{V}$ and $\mathbf{H}$ are given as

$$U_{IJ} = \int_{\Gamma_I} \phi_J^s v_I(Q)\,\mathrm{d}\Gamma \tag{9}$$

$$V_{IJ} = \int_{\Gamma_I} q_J^s v_I(Q)\,\mathrm{d}\Gamma \tag{10}$$

$$H_{IJ} = \int_{\Gamma_I} \Phi_J(\mathbf{s}) v_I(Q)\,\mathrm{d}\Gamma \tag{11}$$

where $v_I(Q)$ is a weight function, Γ_I is a regularly shaped local region around node $\mathbf{s}_I$ in the parametric representation space of the boundary surface. Therefore, the integrals in Eqs. (9)–(11) can be computed without boundary elements.

To assemble Eqs. (7) and (8) into an overall system of equation for the entire domain later, the boundary nodes of subdomain-i can be sorted into n groups: group i containing nodes that belong exclusively in subdomain-i and group j containing nodes that are on the interface with subdomain-j, where j is from 1 to n and $j \neq i$. Accordingly, Eqs. (7) and (8) are partitioned into blocked matrix equations as

$$\begin{bmatrix} \mathbf{U}_{11}^i & \mathbf{U}_{12}^i & \cdots & \mathbf{U}_{1n}^i \\ \mathbf{U}_{21}^i & \mathbf{U}_{22}^i & \cdots & \mathbf{U}_{2n}^i \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{U}_{n1}^i & \mathbf{U}_{n2}^i & \cdots & \mathbf{U}_{nn}^i \end{bmatrix} \begin{Bmatrix} \mathbf{x}_1^i \\ \mathbf{x}_2^i \\ \vdots \\ \mathbf{x}_n^i \end{Bmatrix} = \begin{Bmatrix} \mathbf{H}_1^i\hat{\boldsymbol{\phi}}_1^i \\ \mathbf{H}_2^i\hat{\boldsymbol{\phi}}_2^i \\ \vdots \\ \mathbf{H}_n^i\hat{\boldsymbol{\phi}}_n^i \end{Bmatrix} \tag{12}$$