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Study on natural convection in a cold square enclosure with a pair of hot horizontal cylinders positioned at different vertical locations

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ABSTRACT

A numerical study was conducted to investigate natural convection induced by a temperature difference between a cold outer square enclosure and two hot inner circular cylinders. The immersed boundary method (IBM) based on the finite volume method was used to simulate a two-dimensional natural convection for Rayleigh numbers in the range $10^3 \leqslant Ra \leqslant 10^6$ in the presence of the two cylinders in the cold enclosure. The Prandtl number Pr was taken to be 0.7 corresponding to that of air. This study investigated the effects of the locations of the two cylinders in the enclosure on the heat transfer and laminar fluid flow, when they move vertically along the centerline of the left and right halves of the enclosure. The bifurcation of natural convection from the steady to the unsteady state depended on the Rayleigh number (Ra) and the dimensionless vertical distance from the square cylinder center to the circular cylinder center (δ) of the two cylinders. When $10^3 \leqslant Ra \leqslant 10^5$, the flow and thermal fields eventually reached steady state. However, the state of flow and thermal fields became unsteady for $0 \leqslant \delta \leqslant 0.1$ at $Ra = 10^6$. The dependence of the Nusselt number on Ra and δ was also evaluated.

1. Introduction

Natural convection in an enclosure has practical relevance to several engineering applications such as cooling of electronic equipment, solar collectors, nuclear safety systems, chemical reactors, heat exchangers, and refrigerator condensers. Most engineers in industrial settings want to avoid the use of active control equipment (e.g., fans) for cooling or heating because of additional cost, noise, and vibrational problems. Therefore, it is important to fully understand the mechanism of natural convection.

For several decades, many natural convection studies on the effect of a single cylinder under various thermal conditions in a square enclosure have been conducted both experimentally and numerically [1–11].

Kim et al. [12] investigated a natural convection induced by a temperature difference between a cold outer square enclosure and a hot inner circular cylinder for different Rayleigh numbers varying over the range $10^3 \leqslant Ra \leqslant 10^6$. The location of the inner circular cylinder (δ) was changed vertically along the centerline of the square enclosure. The number, size, and formation of the cell strongly depended on the Rayleigh number and the position of the inner circular cylinder. Under the same conditions, Yoon et al. [13] studied natural convection at $Ra = 10^7$. The natural convection bifurcated from unsteady to steady state according to δ. The thermal and flow fields became an unsteady state in the range of $\delta \leqslant 0.05$ and $\delta \geqslant 0.18$.

Lee et al. [14] carried out a numerical investigation of natural convection in a square enclosure with a circular cylinder at different horizontal and diagonal locations for different Rayleigh numbers varying in the range $10^3 \leqslant Ra \leqslant 10^6$. The distribution of the Nusselt numbers along the cylinder and enclosure surfaces depended on both natural convection induced by the temperature differences between the cylinder and the enclosure, and the gap between the cylinder and the enclosure.

However, natural convection in the presence of an array of cylinders inside the enclosure is quite different from that in the presence of a single cylinder in the enclosure. This is due to the mutual interaction of the buoyant plumes generated by an array of cylinders in addition to the interaction between an array of cylinders and the enclosure. Many experimental and numerical studies have investigated the effect of the presence of an array of cylinders on natural convection in free space [15–24] or in an enclosure [25–28].

Sadeghipour and Asheghi [17] experimentally investigated the steady-state free convection heat transfer from a vertical array of isothermal cylinders numbering from 2 to 8 for Rayleigh numbers of 500, 600, and 700. They investigated the effects of Rayleigh number and cylinder-to-cylinder separation distance on the fluid flow and heat transfer behavior of the cylinders. When the vertical

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Nomenclature

f_i	momentum forcing
g	acceleration of gravity [m/s^2]
L	length of square enclosure [m]
n	normal direction to the wall
$Nu_{left\,cyl}$	local Nusselt number along the left inner circular cylinder
$Nu_{right\,cyl}$	local Nusselt number along the right inner circular cylinder
Nu_{en}	local Nusselt number along the walls of the enclosure
$\overline{Nu}_{left\,cyl}$	surface-averaged Nusselt number along the left inner circular cylinder
$\overline{Nu}_{right\,cyl}$	surface-averaged Nusselt number along the right inner circular cylinder
$\overline{Nu}_T$	surface-averaged Nusselt number along the top wall of the enclosure
$\overline{Nu}_B$	surface-averaged Nusselt number along the bottom wall of the enclosure
$\overline{Nu}_S$	surface-averaged Nusselt number along the side wall of the enclosure
$\overline{Nu}_{en}$	surface-averaged Nusselt number along the square enclosure
P^*	pressure [Pa]
P	dimensionless pressure $\left(=\frac{P^*L^2}{\rho\alpha^2}\right)$
Pr	Prandtl number ($=\nu/\alpha$)
r	dimensionless radius of the cylinder ($=R/L$)
R	radius of circular cylinder [m]
Ra	Rayleigh number $\left(=\frac{g\beta L^3(T_h-T_c)}{\nu\alpha}\right)$
t^*	time [s]
t	dimensionless time $\left(=\frac{t^*\alpha}{L^2}\right)$
T	dimensional temperature [K]
T_h	hot temperature [K]
T_c	cold temperature [K]
u_i^*	velocity [m/s]
u_i	dimensionless velocity $\left(=\frac{u_i^*L}{\alpha}\right)$
x_i^*	Cartesian coordinates [m]
x_i	dimensionless Cartesian coordinates $\left(=\frac{x_i^*}{L}\right)$
Greek symbols	
α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient [K^{-1}]
δ^*	vertical distance from the square cylinder center to the circular [m]
δ	dimensionless vertical distance from the square cylinder center to the circular cylinder center $\left(=\frac{\delta^*}{L}\right)$
δ_{i2}	Kronecker delta
ρ	density [kg/m^3]
ν	kinematic viscosity [m^2/s]
φ	angle from the top of the circular cylinder
θ	dimensionless temperature $\left(=\frac{T-T_c}{T_h-T_c}\right)$
Subscripts/superscripts	
$*$	dimensional value
–	surface-averaged quantity

array of isothermal hot cylinders had the same temperature, the heat transfer coefficient for the bottom cylinder among an array of cylinders had the same value as that for a single hot cylinder when it is within a cold enclosure. However, the heat transfer rate from the other cylinders was smaller than that from a single cylinder when the separation distance from the bottom cylinder to the other cylinders was small, and this followed by an increase in its value with increasing separation distances from the bottom cylinder toward a constant value.

Corcione [22] numerically studied the steady-state free convection from a pair of isothermal cylinders that were arranged horizontally or vertically in free air, for Rayleigh numbers in the range $10^2 \leqslant Ra \leqslant 10^4$, center-to-center horizontal distances ranging from 1.4 to 24, and center-to-center vertical distances to the cylinder diameter ranging from 2 to 12. It was found that the heat transfer rate from a pair of isothermal cylinders may be larger or smaller than that from a single cylinder, depending on the Rayleigh number, the cylinder location in the array, and the array geometry.

Reymond et al. [26] experimentally investigated the natural convection heat transfer from a pair of vertically aligned horizontal cylinders in water for Rayleigh numbers of 2×10^6, 4×10^6, and 6×10^6, and different cylinder spacings corresponding to 1.5, 2, and 3 cylinder diameters. This study showed that the fluid flow and heat transfer from the lower cylinder were unaffected by the presence of the second upper cylinder, and those from the upper cylinder were also unaffected if the lower cylinder was not heated. However, when both the upper and the lower cylinders were heated, the fluid flow and heat transfer from the upper cylinder interacted with a thermal plume rising from the heated lower cylinder and were significantly affected by the presence of the heated lower cylinder.

Although many researchers have conducted studies on natural convection, little information is available on natural convection processes within a cooled square enclosure containing two hot circular cylinders located at different positions along the center of the left and right halves of the enclosure. In this situation, the flow and heat transfer characteristics are largely affected by the location of the two cylinders and the buoyancy-induced convection at different Rayleigh numbers. The purpose of this study was to examine the effects of the locations of two hot inner circular cylinders in the enclosure and the buoyancy-induced convection on heat transfer and fluid flow in the enclosure when the two circular cylinders are located at different positions along the center of the left and right halves of the enclosure at different Rayleigh numbers.

2. Numerical methodology

In this study, we assumed that radiation effects were negligible. The fluid properties were assumed to be constant except for the density in the buoyancy term, which follows the Boussinesq approximation. The gravitational acceleration acted in the negative y-direction. The immersed boundary method (IBM) was used to handle the two cylinders, which were located at the centers of the left and right halves of the enclosure. The governing equations describing unsteady incompressible viscous flow and thermal fields used in the present study were the continuity, momentum, and energy equations in their non-dimensional forms defined as

$$\frac{\partial u_i}{\partial x_i} - q = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + Pr\frac{\partial^2 u_i}{\partial x_j \partial x_j} + Ra\,Pr\,\theta\delta_{i2} + f_i \tag{2}$$

$$\frac{\partial \theta}{\partial t} + u_j\frac{\partial \theta}{\partial x_j} = \frac{\partial^2 \theta}{\partial x_j \partial x_j} + h \tag{3}$$

The dimensionless variables in the above equations were defined as