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Fluid temperature distribution inside a flat mini-channel: Semi-analytical wall transfer functions and estimation from temperatures of external faces

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ABSTRACT

Modelling fluid flow and heat transfer inside a mini- or micro-channel constitutes a challenge because it requires taking into account many effects that do not occur in traditional macrostructured systems. A semi-analytical heat transfer model that takes into account conduction and advection in the fluid as well as conduction in the solid walls (conjugate heat transfer) of a flat mini-channel is first derived and verified. It is based on Fourier transforms of the temperature and normal flux in the direction of the Poiseuille flow. It allows to relate its bulk temperature $T_b(x)$ to external surface sources by two transfer functions without the use of any internal heat transfer coefficient distribution, whatever the location of these sources. The second part of the paper is devoted to the use of this model in an inverse way, that is to retrieve the $T_b(x)$ distribution starting from the additional observation of the noised synthetic temperature profiles over the external faces of both walls of the channel. Estimations of the average velocity and of the external heat transfer coefficient are first implemented. The temperature and flux distributions over the internal faces of the walls are estimated by an inverse method then, before a reconstruction of the internal bulk temperature profile.

1. Introduction

Minichannels, that is channels whose hydraulic diameters range from a few tenth of micrometers to one millimeter, and where fluid flows and heat transfer occurs, are being increasingly used in many different applications: cooling of microprocessors by compact heat exchangers, removal of the heat produced at the level of the bipolar plates in proton exchange membrane fuel cells or temperature control in injection molding of plastic or composite parts, just to quote a few. Designing such experimental devices requires a sound modelling of heat transfer and fluid flow at length scales that differ from those of traditional macrostructured systems because of at least six phenomena, named (a) to (f) and detailed below.

Presence of the solid walls, whose volume fraction is not negligible in the wall/fluid system modifies heat transfer at the solid wall/fluid flow interface: this corresponds to a "conjugate heat transfer" configuration. No new physical effect needs to be considered but introduction of the volume and thermophysical properties of the wall implies that the traditional Nusselt correlations, which yield the local heat transfer coefficient $h(x)$ for forced convection either for imposed wall temperature or for imposed wall flux, have to be revisited because:

(a) the heat flux distribution at the wall is not always normal to it and axial conduction develops inside the wall, with a direction parallel to the velocity;
(b) the location of the heat source modifes the distribution of the heat transfer coeffcient in the flow direction;
(c) the axial boundary conditions, both in the fluid and in the solid walls are usually ill-defined;
(d) because of the small hydraulic diameters and in the case of very viscous fluid, the velocity in the minichannel can become quite low and the axial diffusion term can not be always neglected with respect to the advection term in the heat equation in the fluid: this happens for low Péclet numbers.

Other effects such as the development of the velocity profile (e) [1,2] or the viscous dissipation heating (f) [3–5] can be mentioned but will not be considered in the present study [6].

Effect (a), axial conduction in the wall [7–9], modifies heat transfer especially near the entrance region. Maranzana et al.

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Nomenclature

c	heat capacity, J/kg K
e	thickness, m
$\boldsymbol{F}$	fluid transfer matrix
$\boldsymbol{H}$	external transfer matrix
h	heat transfer coefficient, W m^{-2} K^{-1}
$\mathcal{J}$	cost function
$2L$	virtual channel length, m
$2l$	channel length, m
M	axial conduction number
Pe	Péclet number
Re	Reynolds number
$\boldsymbol{S}$	solid transfer matrix
$\boldsymbol{S}^*$	scaled sensitivity matrix
T	temperature, K
T_∞	ambient temperature, K
U	mean velocity, m/s
$u(y)$	inlet velocity profile, m/s
x, y	spatial coordinates, m
Greek symbols	
α_n	discrete eigenvalue of order n
λ	thermal conductivity, W/m K
ν	kinematic viscosity, m^2/s
φ	heat flux density in y direction, W/m^2
$\tilde{\varphi}$	heat flux Fourier transform
ρ	density, kg/m^3
superscripts	
T	transposed of a matrix
$\sim$	Fourier transform
subscripts	
a	arithmetic average
b	bulk
c	cold
f	fluid
h	hot
s	solid
wc	internal cold wall
wh	internal hot wall

[10] introduced a dimensionless number M (the "axial conduction number") that quantifies the ratio of the heat flow rates transferred by axial conduction in the wall and advective heat transfer in the flow. They showed that this effect can be neglected as soon as the M number gets lower than a threshold value that depends on the channel geometry and on the thermophysical properties of both the fluid and of the wall material.

The first part of this paper concerns the modelling and verification of the solution of both single phase fluid flow and heat transfer (conduction and advection) in a straight and flat mini-channel whose cross section is characterized by a high aspect ratio (width over thickness of the flowing fluid layer). The velocity profile is a Poiseuille parabolic distribution that is fully developed. A surface heat source distribution, uniform in the width direction but variable in the axial direction is imposed on the external faces of the two solid wall layers, which makes the steady state heat equation in the fluid (advection and diffusion) and in the walls (pure conduction) two-dimensional. These external faces are subjected to a uniform heat transfer coefficient with the external air temperature.

The model provides the temperature solution in the fluid and the solid parts of this system using integral transforms (Fourier transforms in the flow direction) without the assumption of any local internal heat transfer coefficient between the interface temperature and the bulk flow temperature. This model, already presented by Perry et al. [11], which took into account effects (a) and (d) has been extended here to accommodate effects (b) and (c).

This analytical model uses transfer functions between Fourier transforms of both temperature and surface heat sources, instead of a heat transfer coefficient $h(x)$. These do not depend on the position of the heat source and are the singular values of the corresponding matrix linking for example the sampled external temperature distribution to the internal wall heat flux sampled distribution.

Apart from the problem of a pertinent thermal modelling for a minichannel flow, another specific difficulty arises when heat transfer in such a confined system has to be characterized: because of its small dimensions, introduction of a temperature or heat flux sensor, in order to estimate the bulk flow temperature, is always more or less intrusive. This modifies either the local flow velocity and/or the internal temperature and heat flux distributions. It is of course possible to embed thermocouple junctions in the least intrusive way possible inside the walls [12]. Another possibility lies in the measurement of the axial temperature distribution at the outer faces of both solid walls by infrared thermography, in order to retrieve the internal distribution profile.

So, the second part of the paper is devoted to the conception of an inverse method allowing to derive the flux distribution on both wall/liquid flow interfaces as well as the internal bulk temperature distribution starting from the measurement of corresponding external surface temperature distributions. It is decomposed into three stages:

- estimation of the structural parameters of this thermal system (mean velocity and external heat transfer coefficient);
- recovering the temperatures of internal walls and the corresponding wall fluxes from the external temperature distribution and from the external heat transfer coefficient estimated previously;
- retrieving the bulk temperature distribution of the flow in the channel by using a heat balance equation written in Fourier's domain.

The last two stages of this paper correspond to an inverse function estimation problem. In this class of problem, a small amount of noise in the measured data, here the sampled external temperature field, may yield very high errors in the recovered internal wall heat flux distribution, if no specific regularization technique is applied. Here this regularization, the Truncated Singular Value Decomposition [13] of the matrix linking the external temperature to the internal flux is implemented in a natural and explicit way since the higher modes of this modal decomposition are neglected in the reconstruction of the space distribution of the flux.

2. The studied system and its modelling

Let us consider the following system (Fig. 1): a liquid flows in a flat channel of length 2ℓ and of thickness e_f, which is defined by two parallel plates of thicknesses e_1 and e_2. A developed velocity