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Improvement and comparison of some estimators dedicated to thermal diffusivity estimation of orthotropic materials with the 3D-flash method

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ABSTRACT

This theoretical and numerical study deals with the estimation of thermal diffusivities of orthotropic materials with the 3D-laser-flash method. This method consists in applying a short non-uniform heat flux to a sample in order to generate three-dimensional heat transfer. An infrared camera is used to measure the evolution of the temperature field at the front face or at the back face of the sample. An estimation procedure, i.e., an estimator, combines these measurements with an analytical solution of the underlying model in order to estimate unknown parameters, i.e., thermal diffusivities, a heat transfer coefficient and parameters related to the spatial shape of the laser beam. In this work, three estimators inspired from previous work are presented and some improvements are proposed. A fourth estimator is introduced and compared to the previous ones. This comparison is based on theoretical standard deviations of thermal diffusivities. Results show that standard deviations can vary up to a factor of 4 and are minimized by using the fourth procedure.

1. Introduction

Among various existing methods that have been proposed to estimate thermophysical parameters, the flash method is the most popular method of measuring the thermal diffusivity of solids. The first approach was proposed in 1961 by Parker et al. [1] and deals with the estimation of the in-depth diffusivity of a sample. It consists in applying a very short and uniform burst of energy at the front face of a sample in order to generate one dimensional heat transfer. The resulting temperature rise at the back face of the sample is measured. The model corresponding to the experiment shows that the in-depth diffusivity can be computed once the half-rise time $t_{1/2}$ is estimated. The experiment, the model and the estimation procedure were later improved. In particular, some developments were devoted to thermally orthotropic materials. Some methods closely related to the original flash method use non-uniform thermal excitations to estimate in-depth and in-plane diffusivities [2–4].

In 1995, Philippi et al. [5] proposed a new experimental setup dedicated to orthotropic materials. This method, called later "3D flash method", allows the three thermal diffusivities of such materials to be estimated. The key idea is to apply a non-uniform burst of energy (from flash lamps or more recently from lasers) on a sample in order to generate three-dimensional heat transfer. An infrared camera is used to measure temperature fields either at the front face or at the back face of the sample. Temperature fields are then used by an estimator to get an estimation of the three thermal diffusivities (Fig. 1). One interesting feature of this method is that it avoids some experimental precautions. Indeed, no knowledge of the spatial shape of the thermal excitation is required. Information about the time shape is not necessary as long as it is short enough so that it can be considered as an impulse excitation (Dirac). Thanks to the use of infrared camera, the issue of sensor positioning disappeared and is replaced by an image calibration.

By modifying the experimental setup of the 3D flash method, Remy et al. [6] proposed a method that estimates the in-plane diffusivity without any knowledge of the spatial and time shape of the thermal excitation.

The 3D-flash method is not the only one that can be used to estimate thermal properties of orthotropic materials. Approaches inspired from hot plates methods [7–9] or based on the 3ω-approach [10] can be used as well. However, in our case, the use of the flash method is motivated by the fact that it is not an intrusive method and that it is suitable for flat or slightly curved surfaces.

Estimating thermal diffusivities with the 3D flash method consists in solving a parameter estimation problem. It involves simultaneously an experiment, a model and an estimation procedure (Fig. 1). The experiment setup provides measurements which consist in a set of temperature fields $T^*_{i,j}(z, t_k)$ at the front ($z = 0$) or at the back face ($z = l_z$) of a sample. A heat transfer model is developed

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Nomenclature

Latin symbols

a_x, a_y, a_z	thermal diffusivity along the x- and y-axis
$A(t_k)$	covariance matrix of observables at time t_k (ENH estimator)
$B_{m,n}(x,y)$	components of the 2D Fourier-cosine basis
$C(z,t_1,t_2)$	unknown constant value associated with the pair of instants $(t_1;t_2)$ (ERH estimator)
$C_{m,n}$	unknown constant value associated with harmonic $(m; n)$ (ENH estimator)
d	diameter of the laser beam
$D(t)$	uniform drift of temperature fields
e_Y	random vector corresponding to the effect of measurement noise on Y^*
$e_{m,n}$	random value corresponding to the noise of harmonic $(m; n)$
e_β	vector of errors on unknown parameters β (dim N_β)
e_γ	vector of errors on a-priori known parameters γ (dim N_γ)
$E_{m,n}$	limit temperature associated with the harmonic $(m;n)$
h	heat loss coefficient
H	Biot number
$J(\beta)$	objective function
l_x, l_y, l_z	dimensions of the sample (thermal images) along the x-, y- and z-axis (m)
N_β	number of unknown parameters
N_γ	number of a-priori known parameters
N_{obs}	number of observables
$NUD(x,y,t)$	non-uniform drift of temperature fields
$N_{\Delta t}$	number of pairs of instants $(t_{1,i}; t_{2,\ i})$
N_t	number of instants t_k
Q	amount of energy absorbed by the sample
p	Laplace variable
$r(x,y)$	shape function associated with the laser beam
$r_{m,n}$	$=r(\alpha_m,\beta_n)$ Fourier coefficients of shape function $r(x,y)$
$r^{\text{ref}}_{p,q}$	Fourier coefficients of the reference harmonic $(p;q)$ of shape function $r(x,y)$
S	selection matrix
t_f	time of the last temperature field
$T(x,y,z,t)$	analytical solution of the heat transfer modeling
$T^*_{i,j}(z,t_k)$	pixel (i,j) at time t_k of the corresponding temperature field
$\overline{T^*(z,t_k)}$	average temperature computed with measurements
$\overline{T(z,t_k)}$	average temperature computed with model outputs
$u(t)$	time shape function of the laser beam
$u(p)$	Laplace transform of time shape function
u_k	strictly positive roots of the transcendental equation (Eq. (19))
$V(\beta)$	likelihood function
W	inverse of the covariance matrix cov(e_Y) (dim $N_{\text{obs}} \times N_{\text{obs}}$)
X or X_β	sensitivity matrix with respect to β (dim $N_{\text{obs}} \times N_\beta$)
X_γ	sensitivity matrix with respect to γ (dim $N_{\text{obs}} \times N_\gamma$)
$X_\beta^{(m)}$	sensitivity matrix with respect to β related to estimation of $\tau_x^{(m)}$ (MSEH estimator)
$X_m(x)$, $Y_n(y)$	solutions of the eigenfunction problem
$y_h(z,t)$	unitary solution of the 1D heat conduction problem in a homogenous wall with third type boundary conditions
Y	vector of model outputs (dim N_{obs})
$Y_{m,n}(z,t_1,t_2)$	a component of observable vector Y (ERH estimator)
$Y_{m,n}(z,t_k)$	a component of observable vector Y (ENH estimator)
Y^*	vector containing "experimental" observables (dim N_{obs})
$Z_k(z)$	solutions of the eigenfunction problem

Greek symbols

α_m, β_n	pulsations along x- and y-axis associated with harmonic (m;n)
β	vector of unknown parameters (dim N_β)
$\hat{\beta}$	vector of estimated parameters (dim N_β)
$\dot{\beta}$	vector containing exact values of unknown parameters (dim N_β)
γ	a priori known parameters (dim N_γ)
$\dot{\gamma}$	vector containing exact values of a-priori known parameters (dim N_γ)
$\delta(k)$	Kronecker symbol
$\delta(t)$	Dirac function
$\theta_{m,n}(z,p)$	Laplace transform of harmonic $(m;n)$, i.e., components of the temperature field in spatial basis $B_{m,n}(x,y)$
$\theta_{m,n}(z,t)$	harmonic $(m;n)$ in spatial basis $B_{m,n}(x,y)$
$\theta^{\text{ref}}_{p,q}(z,t)$	reference harmonic $(p;q)$ (ENH estimator)
$\lambda_x, \lambda_y, \lambda_z$	thermal conductivity along the x-, y- and z-axis
ρC	volumetric thermal capacity
σ_m	standard deviation related to the measurement noise of pixels
$\sigma_{m,n}$	standard deviation related to the noise of harmonic (m;n)
$\sigma^{\text{ref}}_{p,q}$	standard deviation related to the noise of reference harmonic (p;q)
$\widehat{\tau_x}^{(m)}$	estimation of τ_x performed with harmonic (m;0) (MSEH estimator)
$\widehat{\tau_y}^{(n)}$	estimation of τ_y performed with harmonic (0;n) (MSEH estimator)
$\phi(x,y,t)$	heat flux density absorbed by the sample due to the laser beam
$\chi_{m,n}(z,p)$	Laplace transform of temperature field components in spatial basis $X_m(x)Y_n(y)$

Operators

$E[X]$	expectation value of random vector X
cov(X)	covariance matrix of the random vector X
std(Y)	standard deviation of random variable Y
$\langle f(x)\vert g(x)\rangle$	dot product between function $f(x)$ and $g(x)$

Estimators

ERH	estimation using ratio of harmonics
ENH	estimation using normalization of harmonics
MSEH	multiple step estimation using harmonics
DEH	direct estimation using harmonics

and relates unknown parameters to model outputs. A statistical procedure is then in charge of combining measurements and model outputs to give an estimation $\hat{\beta}$ of the unknown parameters β. In other words, an estimator is defined which estimates one or several parameters involved in the statistical distribution of measurements, i.e., in the model.

During the past decades, improvements were achieved concerning the experiment, the model and the estimator. The objective of this study is to focus on estimators in order to evaluate in what extent this choice influences standard deviations of estimations. The four following methods are presented and some improvements are proposed: