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Dual-scale 3-D approach for modeling radiative heat transfer in fibrous insulations

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ABSTRACT

In this work, a dual-scale computationally-affordable 3-D method is developed to simulate the transfer of radiative heat through fibrous media comprised of fibers with different diameters and orientations. The simulations start by generating a virtual fibrous material with specified microstructural properties and then compute the radiative properties of each fiber (i.e., effective phase function, as well as scattering and absorption coefficients) in the structure using the Mie Scattering theory. Considering independent scattering formulations for our fibrous media (media with high porosities), the radiative properties of the insulation material are computed by summing up the radiative properties of each individual fiber, after transforming the phase function values from the fiber's local 3-D coordinates system to a fixed global coordinates system. The radiative properties of the media are then used in the Radiative Transfer Equation (RTE) equation, an integro-differential equation obtained for computing the attenuation and augmentation of an InfraRed ray's energy as it travels through a fibrous medium. Using the Discrete Ordinate Method (DOM), the RTE is then discretized into a system of twenty four coupled partial differential equations and solved numerically using the FlexPDE program to obtain the rate of heat transfer through the entire thickness of the media. Studying media with different microstructural properties, it was quantitatively shown that increasing solid volume fraction, thickness, or fibers' through-plane orientation increases the rate of heat transfer through insulation. With regard to the role of fiber diameter, it was found that there exists a fiber diameter for which radiation heat transfer through a fibrous media is minimal, ranging between 3 and 10 μm for glass fibers operating in a temperature range of about 340–750 K.

1. Introduction

Fibrous materials are the most common insulations currently on the market, thanks to their low production cost and minimal weight. Fibrous insulations can efficiently suppress the convective mode of heat transfer because of the significant friction the fibers cause against fluid motion, leaving radiation and conduction as the only modes of heat transfer (this can easily be shown by calculating the Rayleigh number as defined by Nield and Bejan [1] for heat transfer between two parallel horizontal plates filled with a porous medium). Fibrous insulations are often composed of glass fibers, polymeric fibers, or mineral wool. Depending on the application (i.e., temperature range and geometrical restrictions), different insulation materials, in terms of both the parent materials and the microstructural parameters (fiber diameter, porosity, fiber orientation...), are needed to efficiently block the transfer of heat. Designing efficient fibrous insulations for a given application requires quantitative knowledge of the role played by each of the above parameters in blocking the transfer of heat [2]. In most reported studies, on the other hand, an insulation material has been treated as a lumped system. The usual outcome of a lumped-model approach is an "effective thermal conductivity" which combines the contributions of each individual parameter in blocking conductive and radiative heat into a single value. Obviously, such an approach does not provide clear guidelines for improving the performance of a product as it does not isolate the source of a problem [3–5].

The complexity of studying heat transfer through a disordered fibrous media lies mostly with the radiation component. This is because porosity of insulation media is normally very high and therefore contribution of the solid phase (i.e., fibers) in heat conduction through the media is negligible compared to that of the interstitial fluid (normally air), unless the fibers are highly conductive (e.g., Aluminum or Steel fibers), which is not very common [2]. Therefore, one can estimate the rate of heat conduction through most insulation media using only the air conductivity and the material's thickness. The radiative component of heat however, varies very differently in insulation media with different fibrous

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Nomenclature

a, b	wave expansion coefficients
C	single fiber cross section
d_f	fiber diameter
$\hat{e}$	unit vector parallel to the fiber axis
E_s	scattered electric field vector
H_s	scattered magnetic field vector
$\tilde{H}_n$	Henkel function of the second kind
i	$\sqrt{-1}$
I	spectral radiative intensity
I_b	spectral blackbody radiative intensity
J_n	integral order Bessel function
K	wave number
m	ratio of refractive index of the fiber and the surrounding medium
M_n	vector cylindrical harmonic
n	index counter
N	number of fibers in the material
N_n	vector cylindrical harmonic
Q	single fiber efficiency
r	fiber radius
$\vec{R}$	unit direction vector
T	scattering amplitude
τ	transmittance: ratio of transmitted radiation heat flux to the incident radiation flux
x	size parameter
β	extinction coefficient
σ	scattering coefficient
ξ	polar angle of incident radiation
ξ_f	through-plane orientation of fiber
κ	absorption coefficient
η	angle of observation in terms of material coordinate system
θ	angle of observation in terms of fiber coordinate system
ϕ	angle of incidence of IR
ϕ_C	half apex angle of scattering cone
Φ	scattering phase function
ψ	scalar wave function
μ	permeability of the medium
ω	angular frequency
λ	wavelength of IR
ε	emissivity
$dC_{sca}/d\Omega$	differential scattering cross-section

Subscripts

f	fiber
sca	scattering
abs	absorption
ext	extinction
i	incident
s	scattered
S	source
C	sink
I	parallel component of light
II	perpendicular component of light

microstructures, especially for media operating at different ranges of temperatures. This study is therefore focused on studying the role of microstructural parameters on the performance of fibrous materials when they are used to insulate radiative heat transfer. To do so, we have developed a dual-scale modeling strategy that allows one to simultaneously study the influence of microscale (the radiative properties of individual fibers like refractive index or fiber orientation) and macroscale (e.g., thickness or porosity) parameters on the performance of an insulation material without requiring excessive computational power. Our microscale calculations are based on the Mie scattering theory. We start our simulations by producing virtual 3-D fibrous structures with specified in-plane and through-plane fiber orientations (mean and standard deviation). We then compute the radiative properties of each fiber within the virtual structures. Assuming independent scattering theory (applicable to media with high porosities), the radiative properties of the whole media is then computed by summing the radiative properties of each individual fiber [6]. These radiative properties are then used in the Radiative Transfer Equation (RTE) to be solved for the heat flux through insulation media with different macroscale properties. With the RTE being a complicated integro-differential equation, different approximate methods have been adopted by the heat transfer community over the past decades to solve this equation numerically [7]. For our macroscale calculations, we considered the Discrete Ordinates Method (DOM) with an S4 approximation.

In the next section, the RTE and our implementation of the DOM-S4 method are described in detail. Our 3-D virtual fibrous media and the application of the Mie scattering theory for obtaining their radiative properties and a validation study are presented in Section 3. Our results and discussion are presented in Section 4, where we discuss the effects of different microstructural parameters of fibrous insulation media on their performance. Our overall conclusions drawn from this work are given in Section 5.

2. Macroscale formulations

An IR beam looses energy as it travels through a fibrous medium due to scattering and absorption, and gains energy due to in-scattering and fiber emission along its path. Equation for conservation of energy along a given path (direction) with which we can tally the change in energy in a given direction for a small wavelength interval is called Radiative Transfer Equation (RTE) [6]

$$\frac{dI}{dS} = -\beta I + \kappa I_b + \frac{\sigma}{4\pi}\int_{\Omega=0}^{4\pi} I(\Omega)\Phi(\Omega)d\Omega \quad (1)$$

The radiative heat that transfers across a fibrous insulation material can be estimated by solving the RTE. In order to obtain the radiative properties, both theoretical and experimental methods have been considered in the past. Inverse determination of the radiative properties of an insulation material has been considered using the heat transmittance data obtained from experiments [8]. The general approach in such studies has been to assume a common form of phase function like (e.g., Henyey–Greenstein phase function) and parametrically vary the scattering and absorption coefficients to make predictions of the RTE match experimental data [9,10].

According to the electromagnetic theory, fiber diameter and temperature are the most important parameters that the treatment of radiative transfer hinges upon. The electromagnetic theory is well established and has been widely used to describe the interaction of an IR ray with cylindrical objects [11,12]. A so-called *size parameter* (x) is therefore defined to relate the radiation wavelength (i.e., temperature) to the fiber diameter [6]: